Towards a better understanding of seasonal climate variability using causal discovery methods

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EGC Conference Dijon – 24 January 2024

Agenda of this lecture

1. **Climate complexity and resulting challenges**

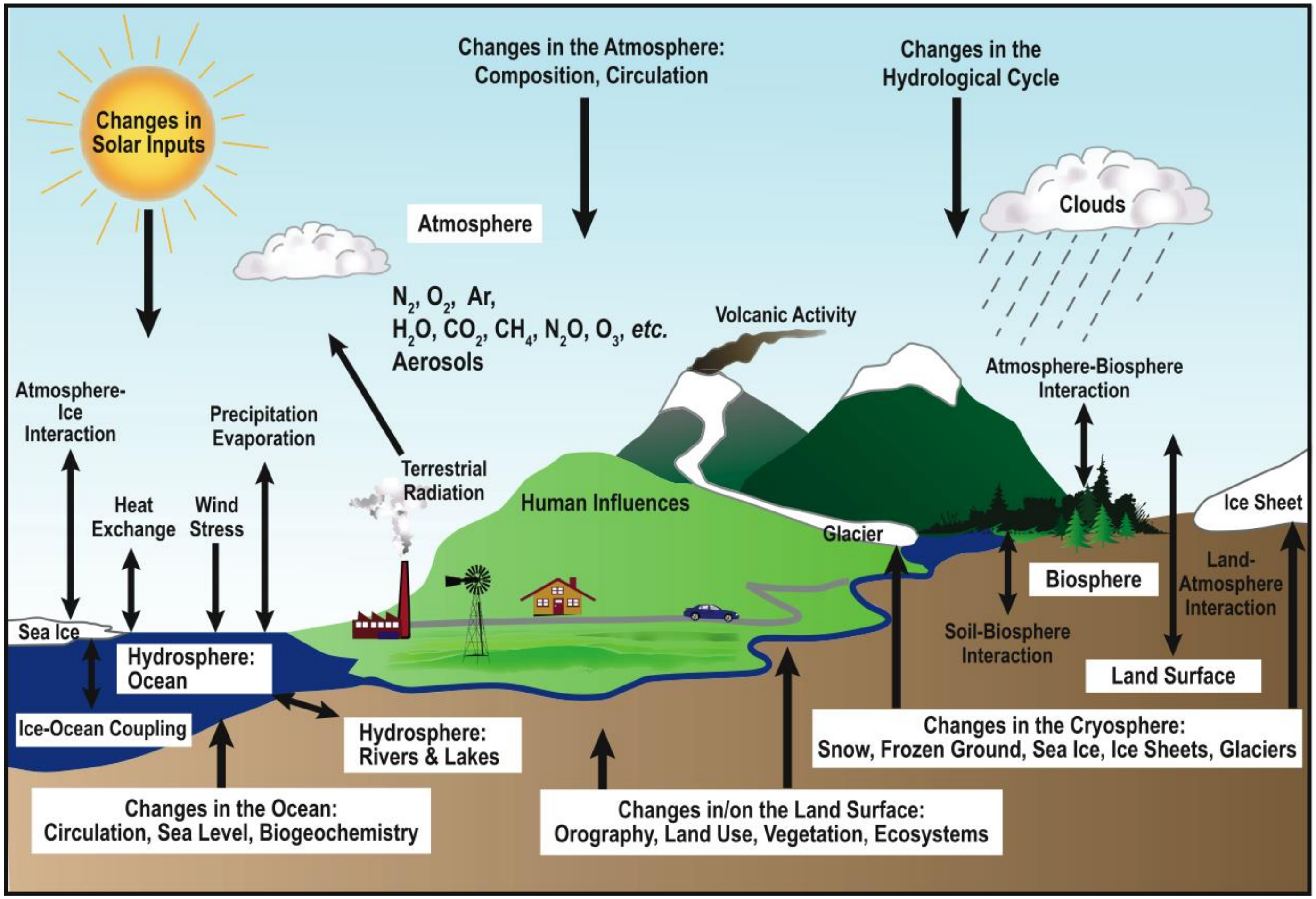
2. **Exploiting climate data: from correlation to causality**

3. **Causal effect networks and applications (mainly) in climate**

4. **Challenges to be addressed**

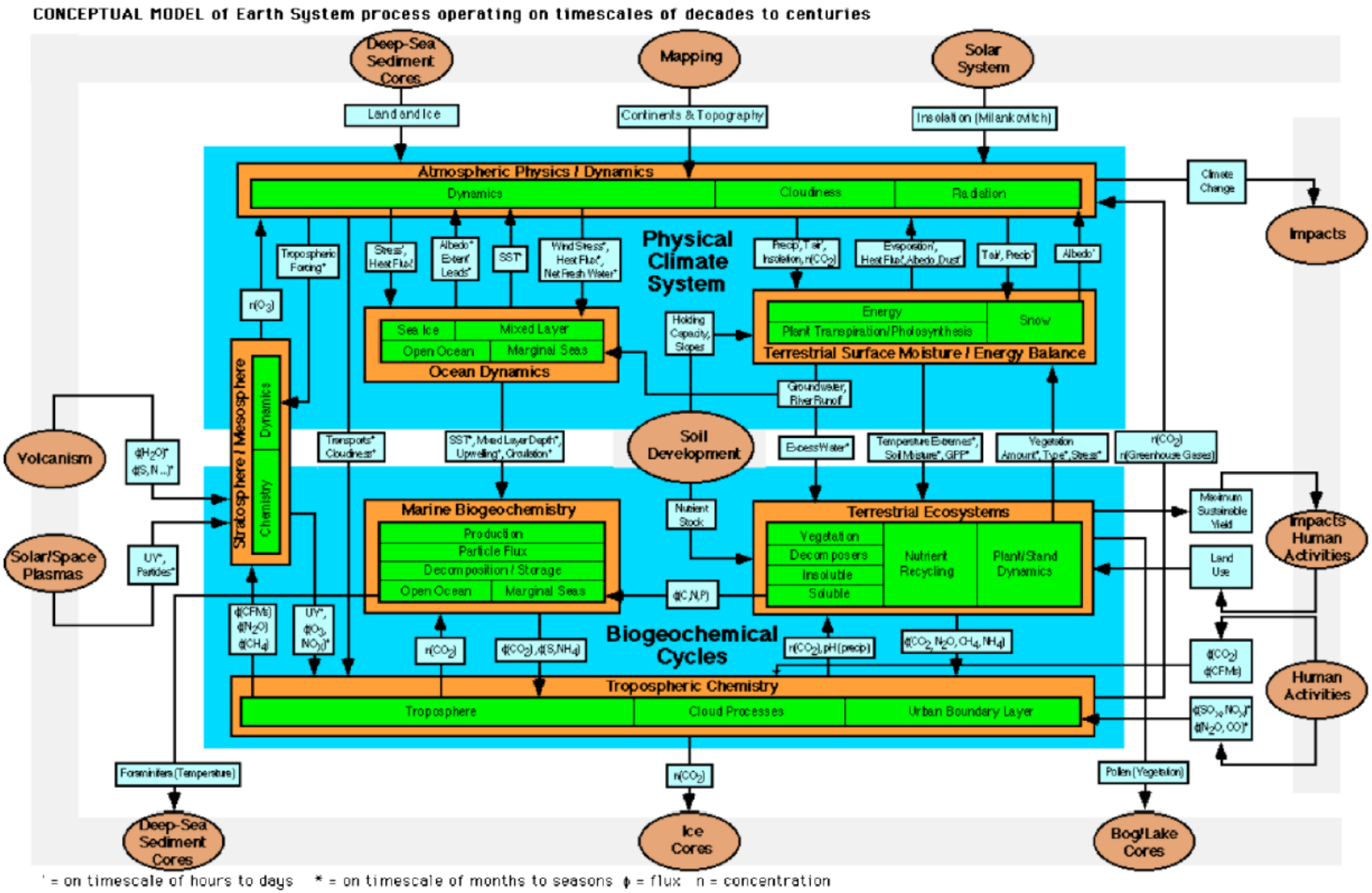
5. **Take home messages**

Climate: A conceptual view



IPCC 4th Assessment Report, 2007

The Bretherton horrendogram (1986)



National Research Council, 1986

Characteristics of the climate system

Complexity: Interplay of multiple individual subsystems

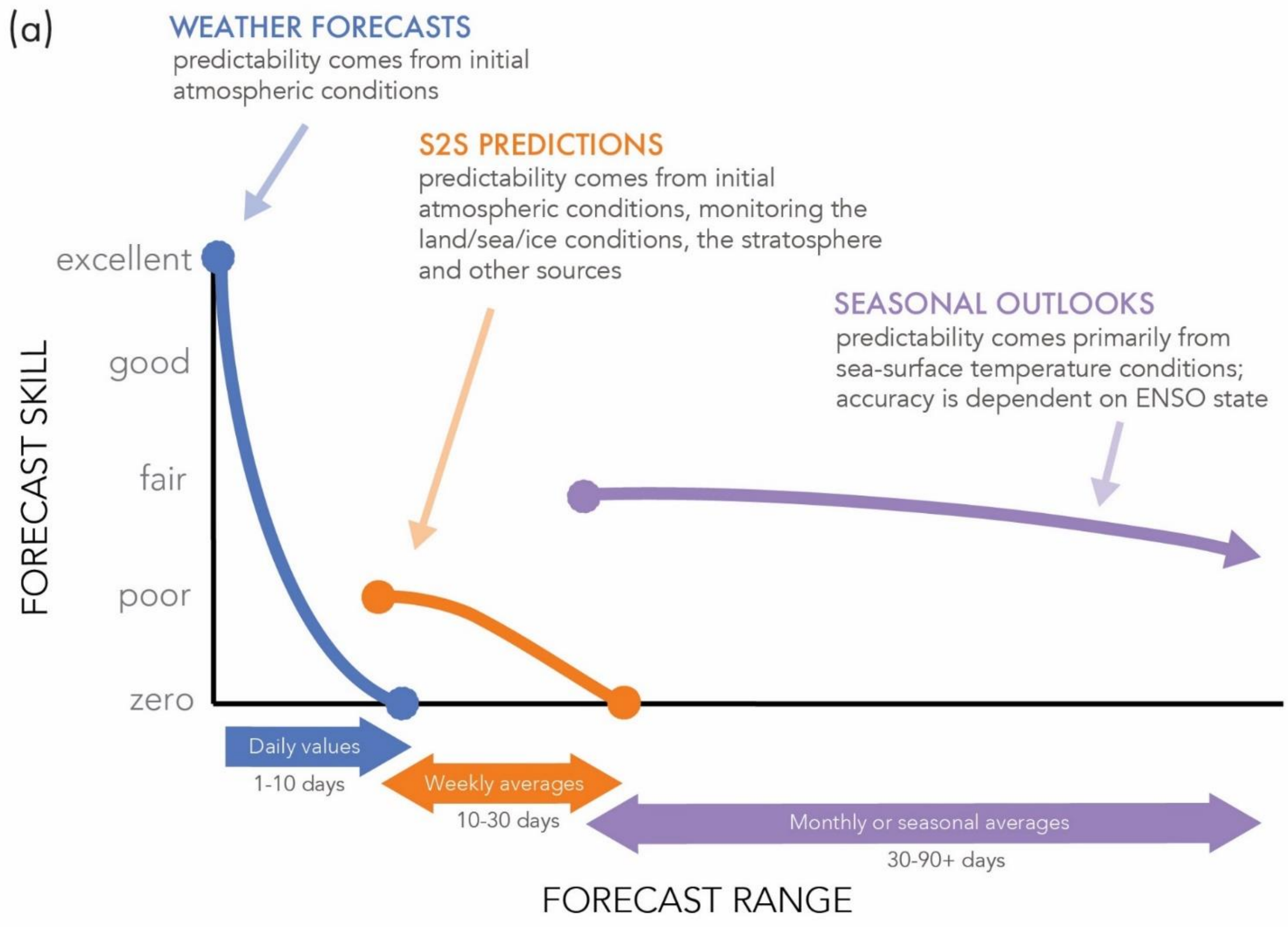
Nonlinearity: Behavior of individual subsystems controlled by mechanisms, for which the response does not depend linearly on the driving variables

$\Rightarrow$ Necessary condition for existence of qualitative different system states

Multi-scale variability: relevant processes take place on a variety of spatial and temporal scales

Nonstationarity: Relevant external parameters are time-dependent (solar insolation, aerosol and greenhouse gas concentrations in atmosphere)

Numerical weather and climate predictions



White et al., 2017

Numerical weather and climate predictions

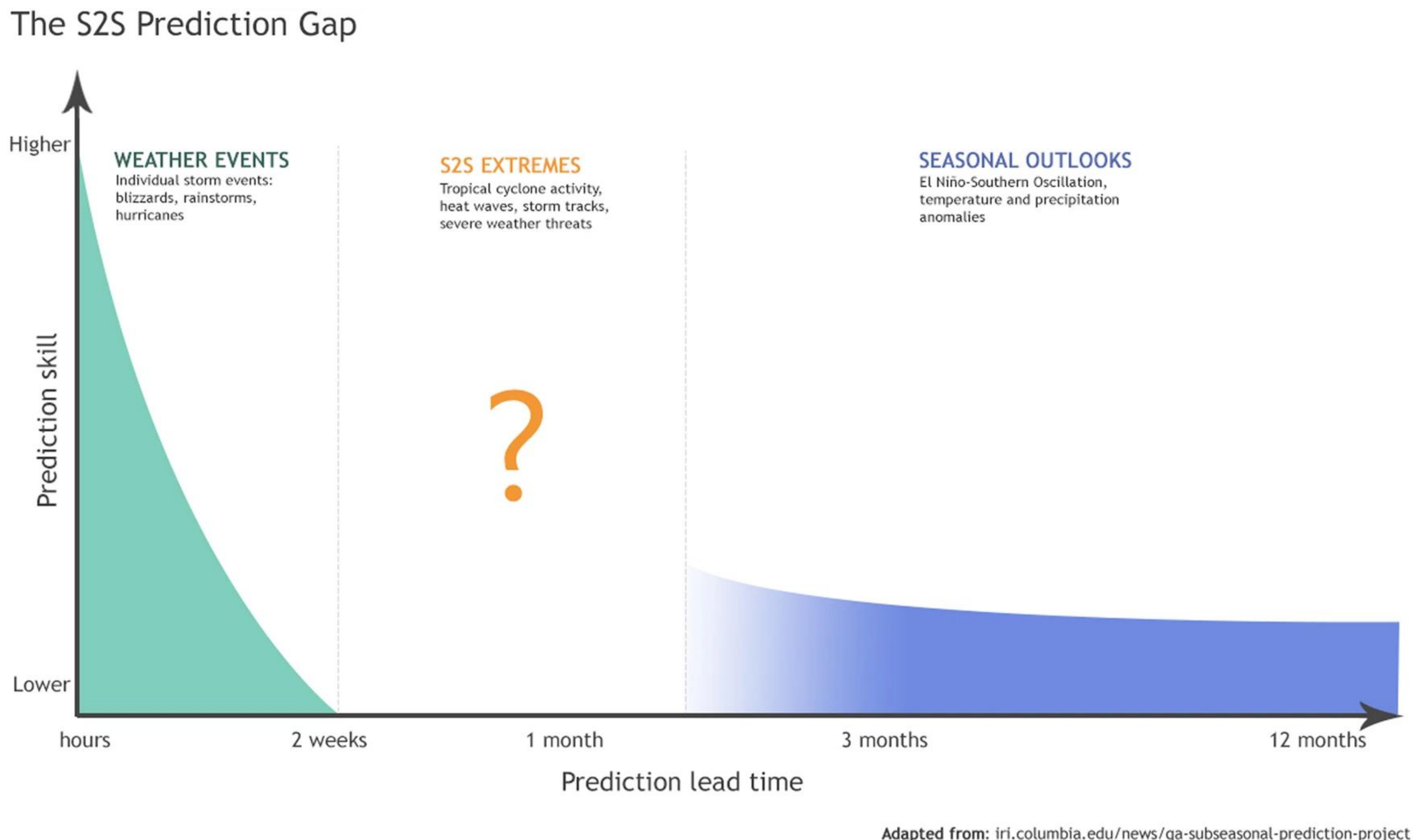
nature > news > article

NEWS | 14 November 2023

DeepMind AI accurately forecasts weather – on a desktop computer

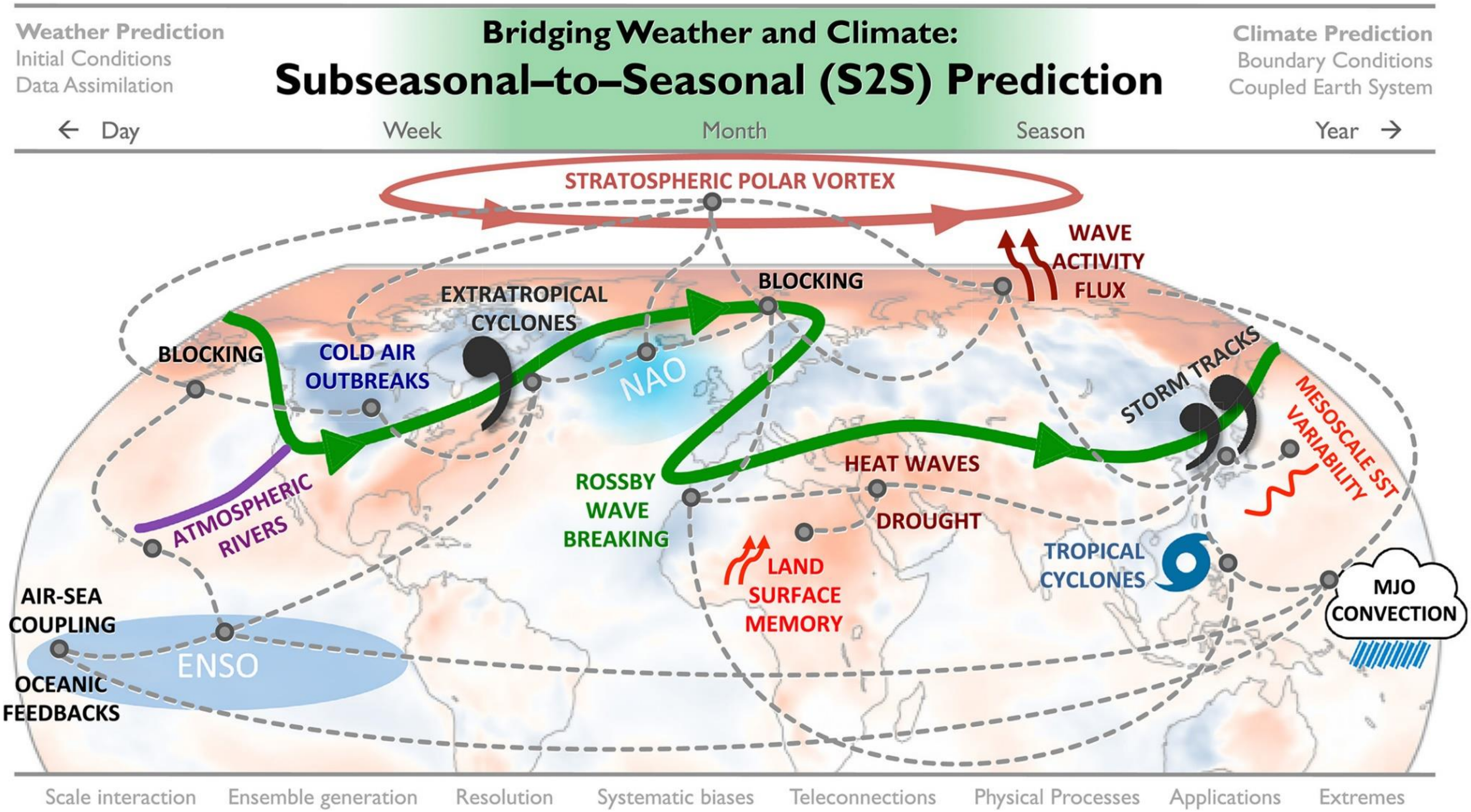
The machine-learning model takes less than a minute to predict future weather worldwide more precisely than other approaches.

Numerical weather and climate predictions



Mariotti et al., 2018

Teleconnections as sources of predictability



Lang et al., 2020

Example: El Niño Southern Oscillation (ENSO)

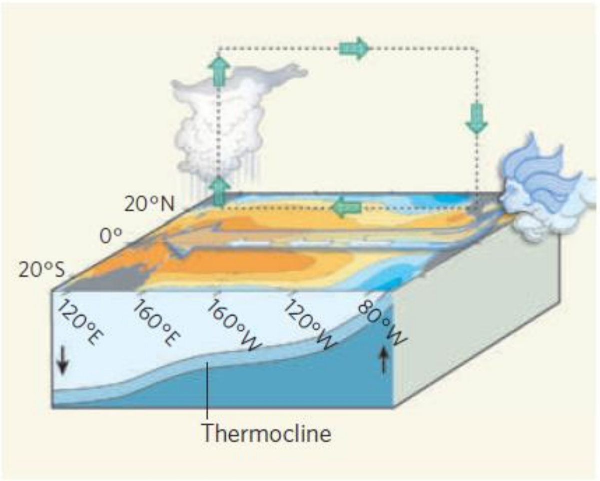
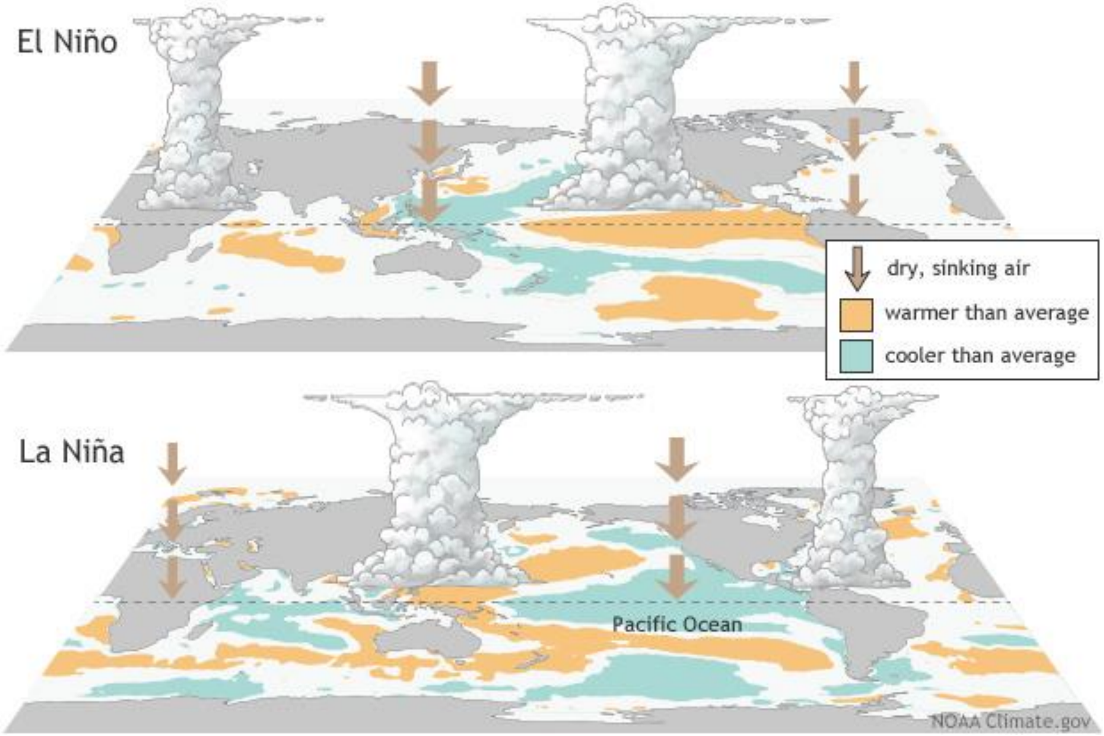


Figure 1 | Normal conditions in the tropical Pacific. Warm surface water and air are pushed to the west by prevailing winds. A consequence is upwelling of cold water on the eastern side, and a shallow thermocline (a subsurface boundary that marks a sharp contrast between warm upper waters and colder deeper waters). Opposite oceanographic conditions prevail on the western side. In the atmosphere, the west is warmer and wetter. Here and in Figure 2, redder colours denote warmer waters, bluer colours denote cooler waters.

Ashok & Yamagata, 2009



www.climate.gov/enso

Overarching question

How to identify teleconnection patterns relevant for a specific climate phenomenon of interest and unveil the underlying physical mechanisms by which they exert control?

EOF analysis

Original motivation: extract dominating variations from spatio-temporal fields of climate observations records

<u>Linear PCA:</u>

Diagonalization of lag-zero covariance matrix C of multivariate time series (matrix *X*)

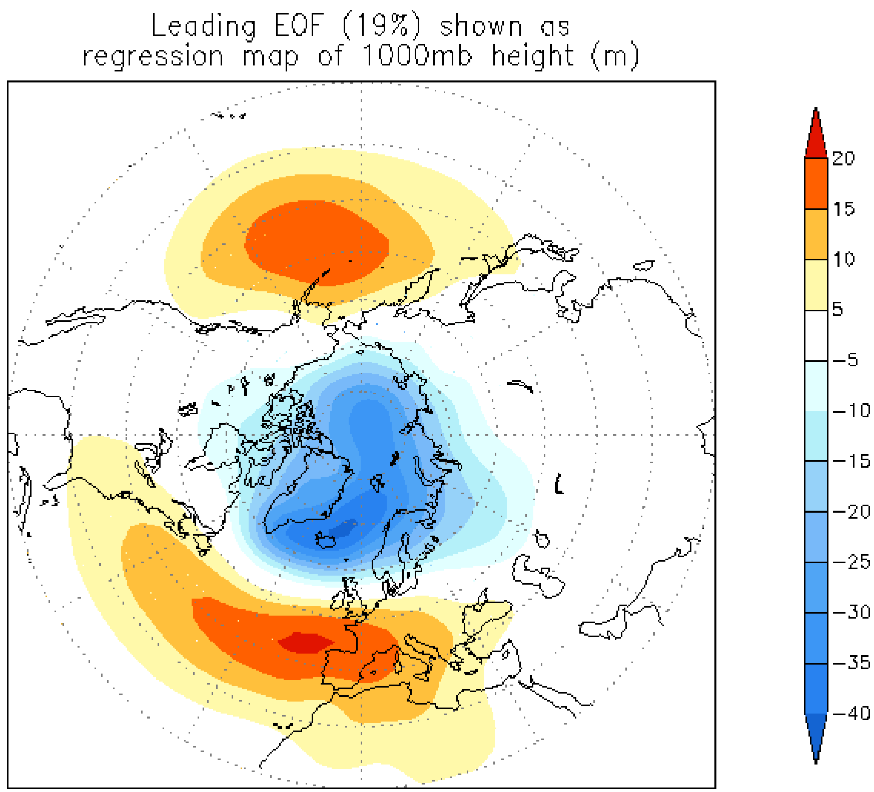
$$C = X^T X \quad \textbf{with} \quad C = U^T \Sigma U \quad \textbf{and} \quad \Sigma = diag(\sigma_1^2, \ldots, \sigma_N^2)$$

- **Compute correlation matrix of all variables**
- **Estimate eigenvalues and eigenvectors**
- **Eigenvectors: additive decomposition into principal components (weighted superpositions of original variables) with individual variances corresponding to associated eigenvalues**

$\Rightarrow$ **spatial EOF patterns + index/score time series describing magnitude and sign of individual EOF modes**

EOF analysis

Example: leading EOF (EOF-1) of near-surface air pressure in Arctic
=> Dipole structure (Arctic Oscillation)



EOF analysis

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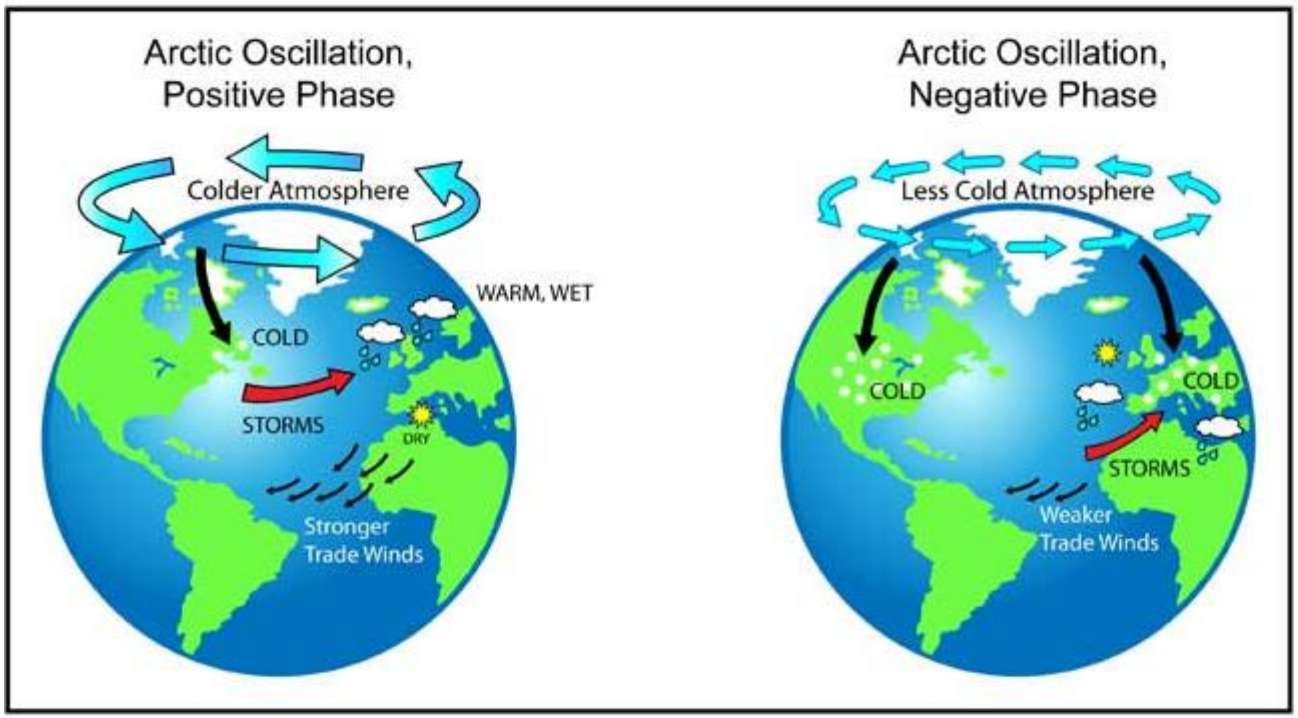


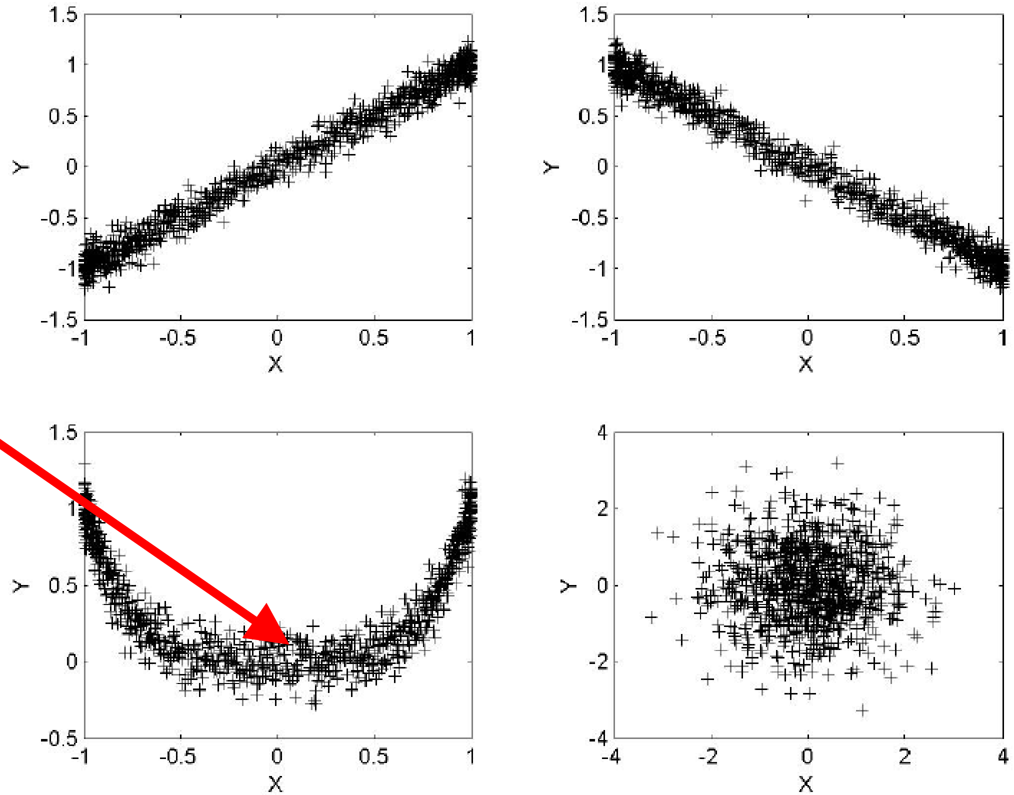
Limitations of EOF analysis

Purpose: extract dominating spatio-temporal (co-)variability modes from fields of climate observations

- **Linear decomposition/dimensionality reduction technique**
- **EOFs modes do not always coincide with specific climatic mechanisms**
- **Intrinsic tendency to exhibit dipole (or multipole) structures enforced by orthogonality constraint between modes**
- **Multiple superimposed patterns need to be considered**
- **Spatial patterns = strength of co-variability, unclear relevance of associated temporal patterns in other regions not highlighted by the same EOF**

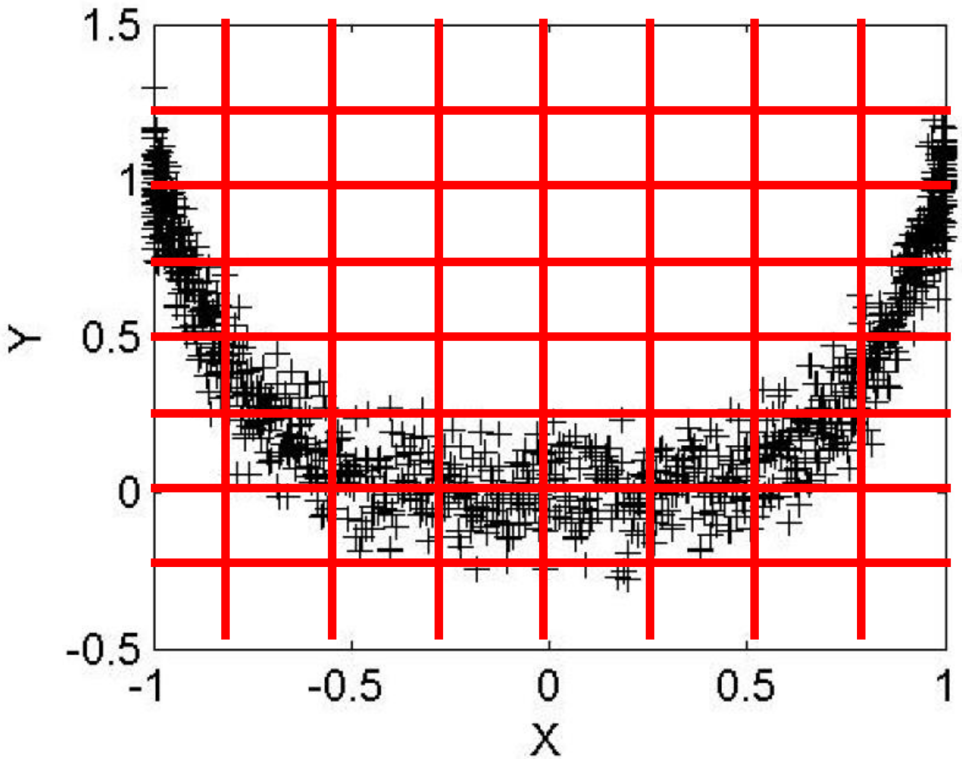
Limits of linear correlations

low correlation coefficient despite clear statistical relationship



Nonlinear dependence: Mutual information

Basic idea: discretization of dynamics into "symbols" and quantification of contingency table of symbol frequencies (joint vs. marginal probabilities)



Nonlinear dependence: Mutual information

Recipe: transform time series into discretized representation using abstract symbols (*a, b)* from discrete (finite) alphabet *A*

$\Rightarrow$ **allows computation of different information-theoretic quantities:**

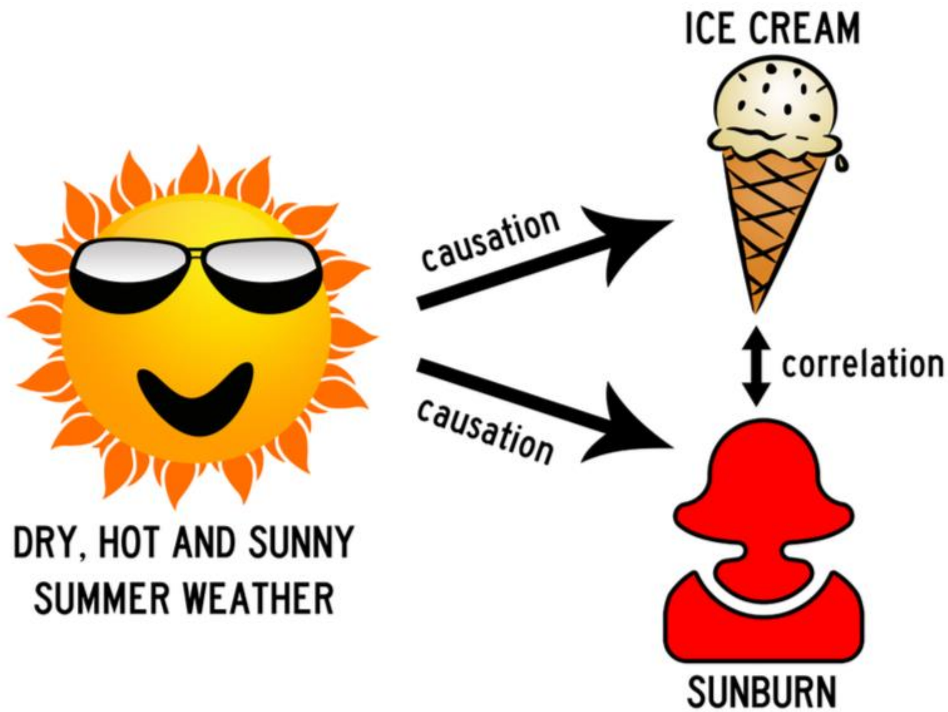
mutual information function (measure for general statistical association)

$$\boxed{I_{XY}(\tau) = \sum_{a,b\in A} P_{ab}^{XY}(\tau) \log_2 \frac{P_{ab}^{XY}(\tau)}{P_a^X P_b^Y}}$$

corresponds to entropy difference

$$I_{XY} = H_X + H_Y - H_{XY}$$

Correlation versus Causality



Correlation (co-variability / statistical dependence) does <u>not</u> imply causality

Causality does not even imply (linear) correlation

Causality among two time series

Classical bivariate approach: linear predictive (Granger) causality

1. **Build linear regression models (bivariate AR models)**

$$X(t) = \sum_{j=1}^{p} A_{XX,j} X(t-j) + \sum_{j=1}^{p} A_{XY,j} Y(t-j) + \varepsilon_X(t)$$
$$Y(t) = \sum_{j=1}^{p} A_{YX,j} X(t-j) + \sum_{j=1}^{p} A_{YY,j} Y(t-j) + \varepsilon_Y(t)$$

2. **Compare variance of error term ε_X (ε_Y) with and without inclusion of *Y* (*X*) in the first (second) equation**
   - **If additional term for *Y* in equation for *X* reduces error: *Y* Granger-causes *X***
   - **If additional term for *X* in equation for *Y* reduces error: *X* Granger-causes *Y***
   - **Practical: are $A_{XY,j}$ ($A_{YX,j}$) significantly different from *0* (e.g., via *F* test)?**

Granger causality ~ predictive skill contributed by another variable

Causality among two time series

Classical Granger causality is based on predictions using a (linear) model for dependences among different instances of two time series

$\Rightarrow$ **Linearity (as well as any other model) assumption can be relieved by information theoretic quantities: conditional mutual information, transfer entropy, etc.**

$$\mathrm{I}(\mathrm{X};\mathrm{Y}\,|Z) = \sum_{z\in Z} p(z) \sum_{x\in X} \sum_{y\in Y} p(x,y|z) \log \frac{p(x,y|z)}{p(x|z)\cdot p(y|z)}$$
$$= H(X|Z) + H(Y|Z) - H(X,Y|Z)$$

- **No conditioning variable *Z*: standard mutual information (nonlinear variant of linear correlation)**
- ***X* = current instance, *Y* = current and past instances, *Z* = past instances of *X*: transfer entropy from *Y* to *X* (nonlinear variant of linear Granger causality)**

Causality among two time series

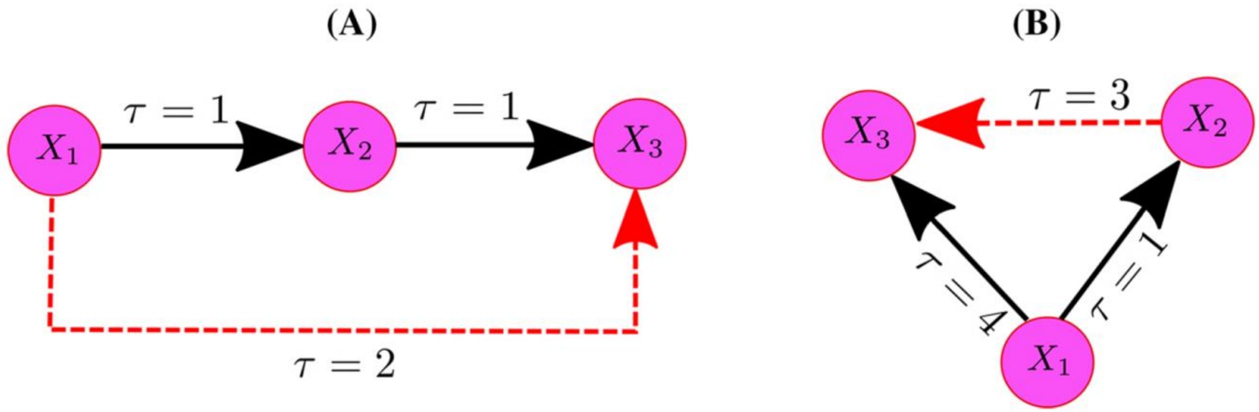
General view: causality is statistically reflected by some directed statistical association between two time series respecting temporal order (cause precedes consequence at a certain lag or set of lags)

Practical challenges (selection):

1. **Linear framework (partial correlations or related characteristics based on linear regression models) too restrictive (may overlook nonlinear dependencies)**
   - $\Rightarrow$ **How do I know if linear framework is sufficient?**
2. **Nonlinear framework often too data-demanding for robust estimation based on density estimation for the explicit time series values**
   - $\Rightarrow$ **Choice of entropy concept/dynamical aspect to focus on**
3. **Identification of relevant time lags to be used (a) for the driver-response relationship between *X* and *Y* and (b) in the conditioning variable(s)**
   - $\Rightarrow$ **Model selection problem**
4. **Effect of any third variables *Z* other than *X* and *Y* on their mutual relationship is not considered**

Causality among multiple time series

Need to distinguish direct from indirect statistical associations mediated by any third variable(s) (e.g. common driver or mediator)

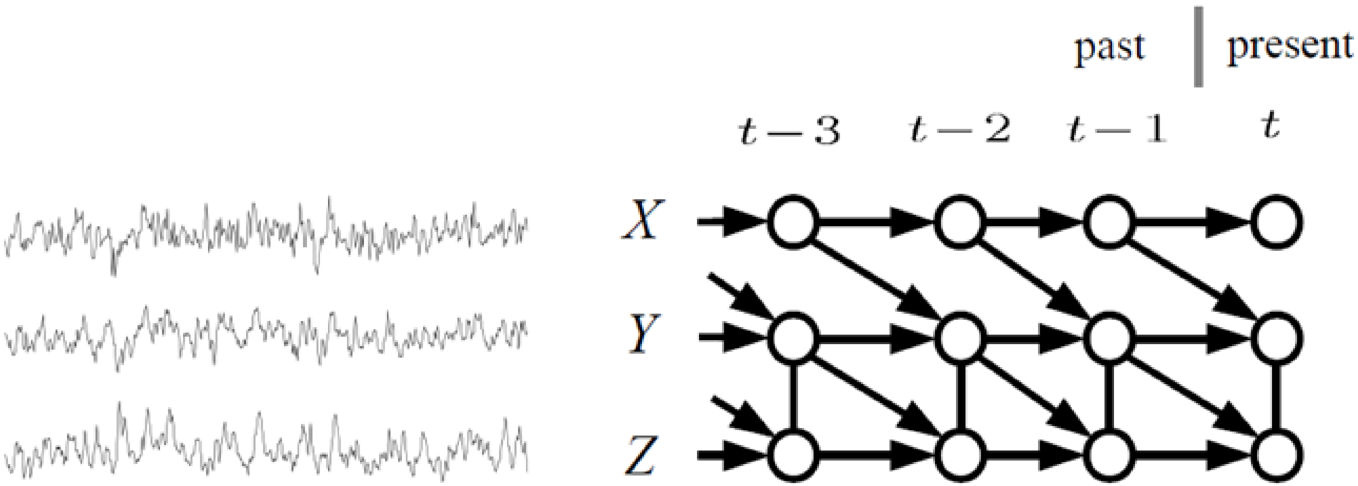


$\Rightarrow$ **Causality between two variables (in the sense of a directed statistical dependence) is <u>always relative</u> to the set of additional variables taken into account for conditioning**

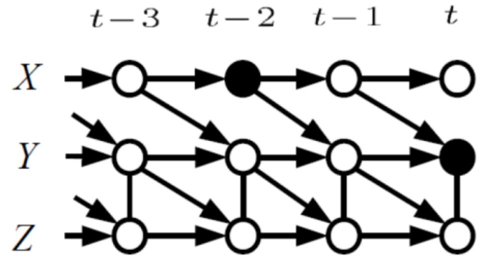
Remark: This further complicates the problem of identifying relevant lags for all variables and also adds to computational complexity and data demand

Causality among multiple time series

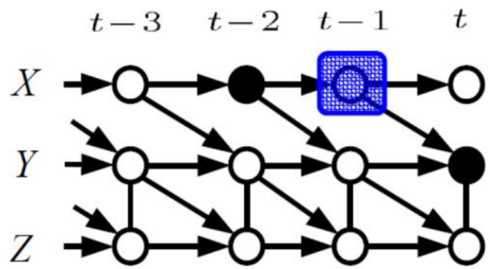
Conceptual framework: Graphical models



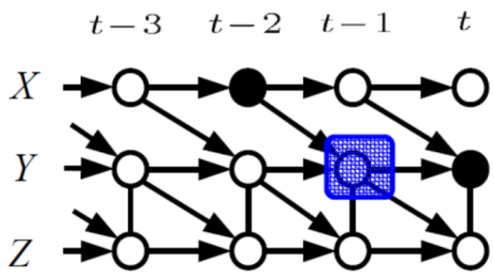
S. L. Lauritzen, Graphical Models, Oxford, 1996
R. Dahlhaus, Metrika 51, 157 (2000)
M. Eichler, Probability Theory and Related Fields 1 (2012)



$$I(X_{t-2}; Y_t) > 0$$



$$I(X_{t-2}; Y_t | X_{t-1}) > 0$$



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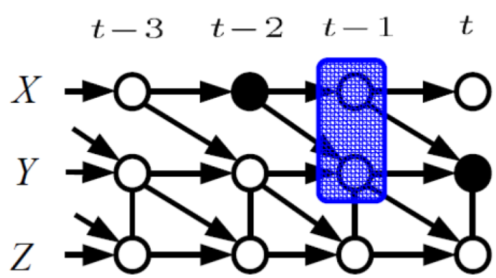


$$I(X_{t-2}; Y_t | Y_{t-1}, X_{t-1}) = 0$$

Causality among multiple time series

Causal inference requires two ingredients:

1. Proper directed statistical association measure

Challenge: robust estimation of conditional probabilities from limited data with a possibly large set of conditioning variables

2. Distinction between direct and indirect associations based on consideration of any possible conditioning factors (serial dependence of response variable and lagged cross-dependency with any third variables, both taken at arbitrary lags)

$\Rightarrow$ **Causal effect network approach (e.g. using Python package tigramite, supporting partial correlations and conditional mutual information)**

Algorithmic gold standard: proper iterative exclusion of conditioning sets by means of successive conditional independence testing (e.g. using the PCMCI algorithm, cf. Runge, Chaos, 2018)

Problem Setting 1

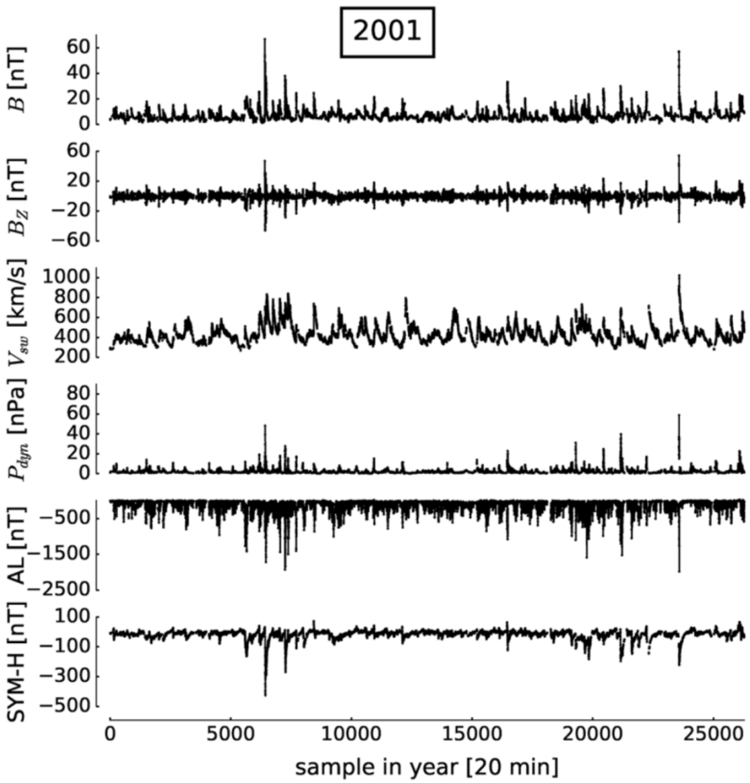
Given:

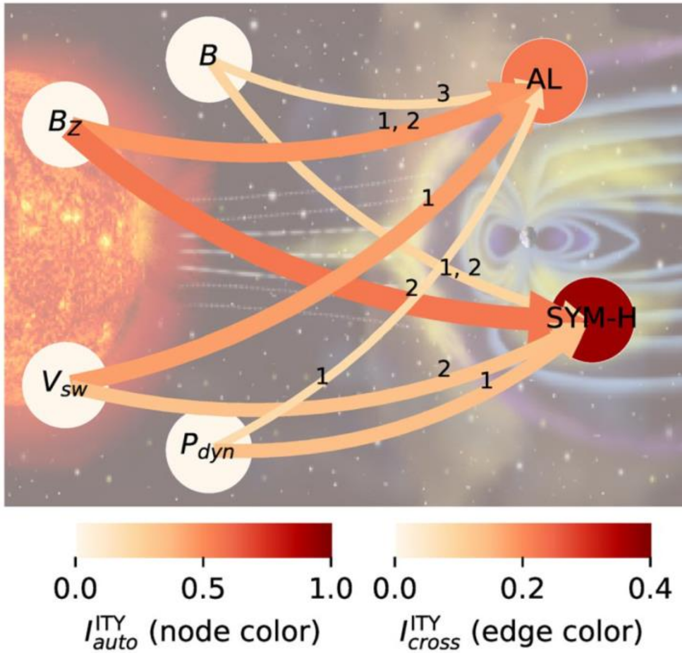
- **a set of time series of different variables supposed to reflect dynamical interactions between different parts of a complex (geo-) system**

Required:

- **Identification of direct vs. indirect coupling between variables**
- **Direction, strength and associated lag(s) for each inferred direct connection**

Example 1: Geomagnetic indices / solar wind

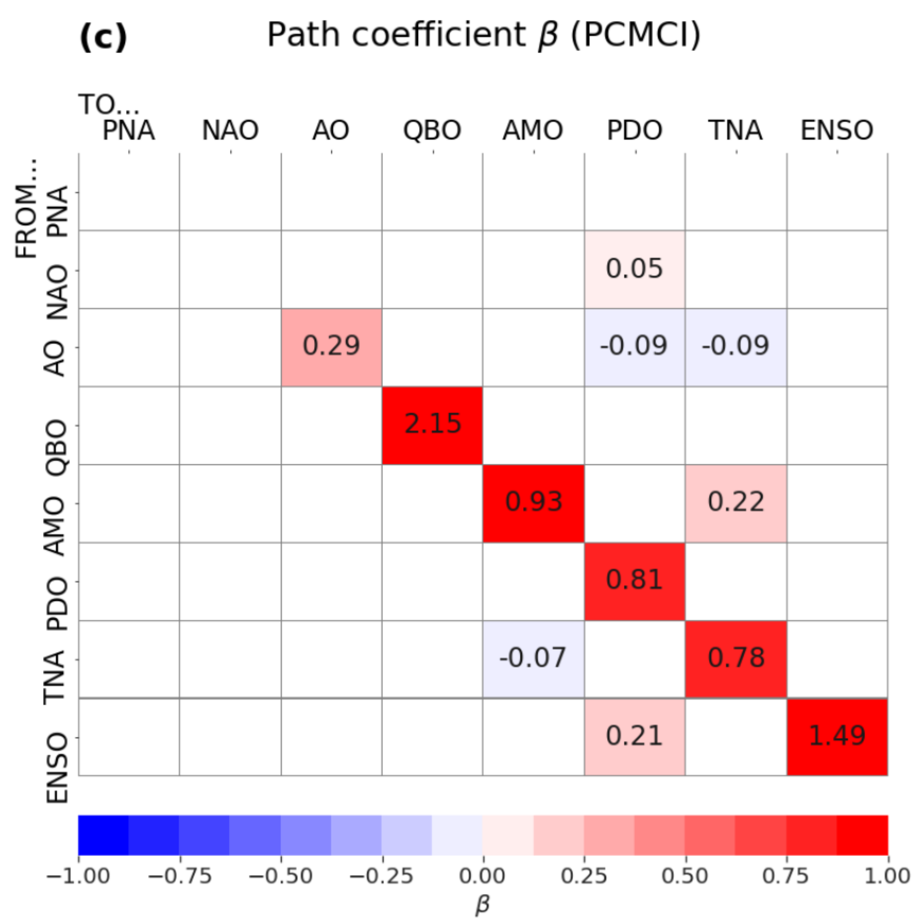




Runge et al., 2018

Complication: causal links may change with time (different states of the coupled solar wind-magnetosphere system) and also time-scale considered

Example 2: Climate indices



Process identification for various teleconnection indices

Docquier et al., in press

Problem Setting 2

Given:

- **One or more target variables to be causally explained and a large set of candidate variables (e.g. spatial field of climate variability) from which relevant precursor/predictor variables need to be identified first**

Required:

- **Identification of direct vs. indirect coupling between variables**
- **Direction, strength and associated lag(s) for each inferred direct connection**
- **Downstream effects of causal drivers: tables/spatial maps indicating which target series are causally affected by the identified set of causal drivers influences (and how strongly)**

General strategy

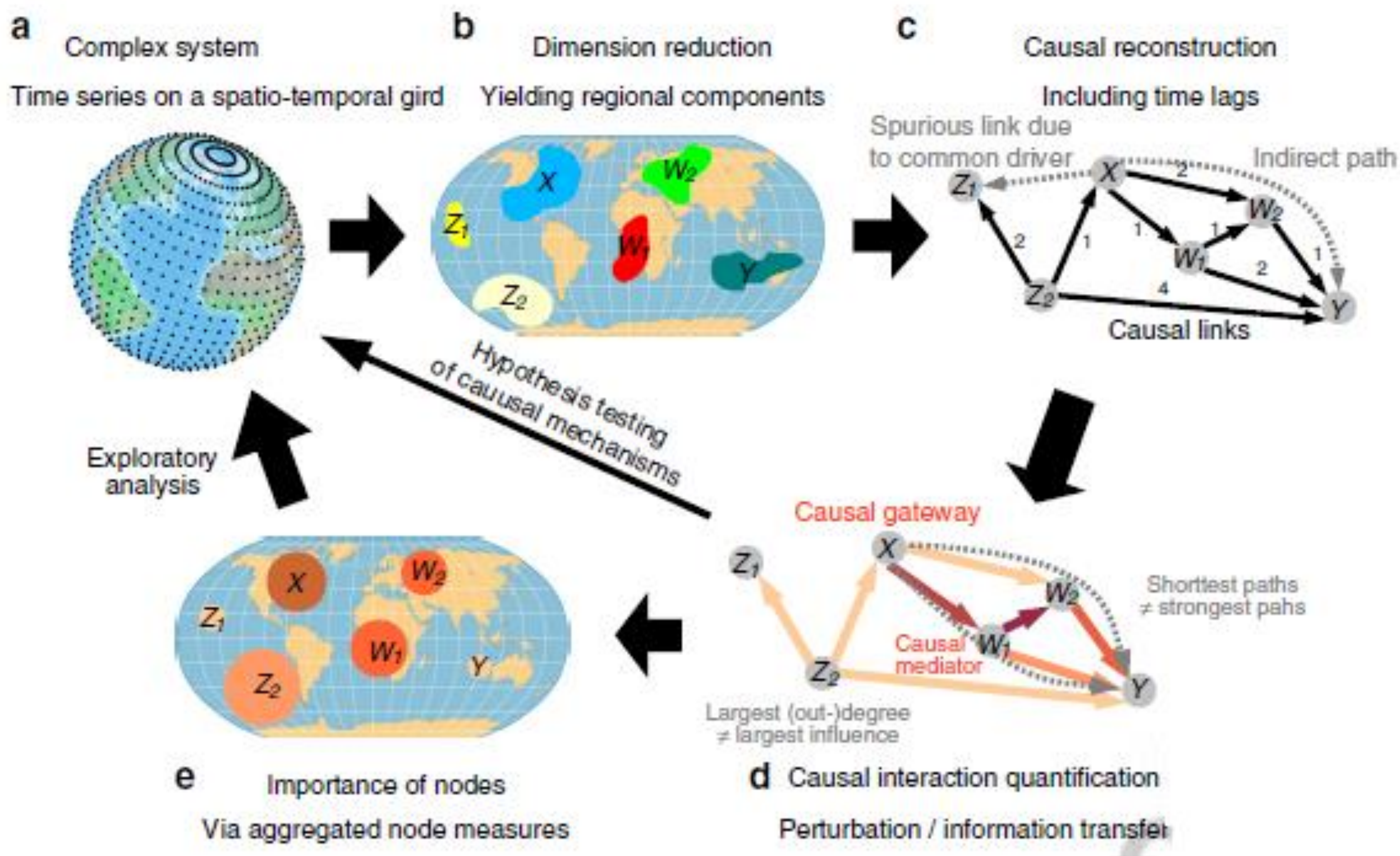
Step 1: Identify relevant actors (variables with associated time series) = network nodes

- **Multivariate statistics (e.g. EOF analysis)**
- **Variables/regions with strong bivariate statistical association with target variable (e.g. correlation maps): response-guided precursor detection**
- **Variables/indices representing key processes/hypotheses discussed in literature: theory-guided precursor selection**

Step 2: Identify causal links <u>among set of considered actors</u> along with their directionality, strength, and time lag

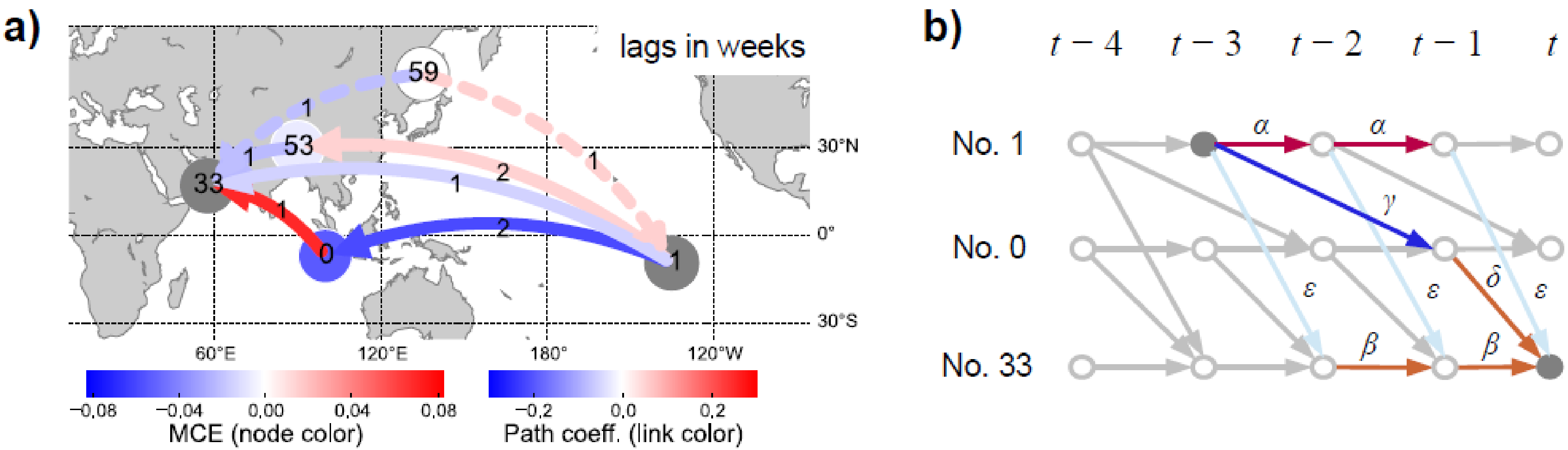
- **Test hypotheses on processes**
- **Identify new teleconnections**
- **Quantify relevance of actors and links**

Idealized workflow



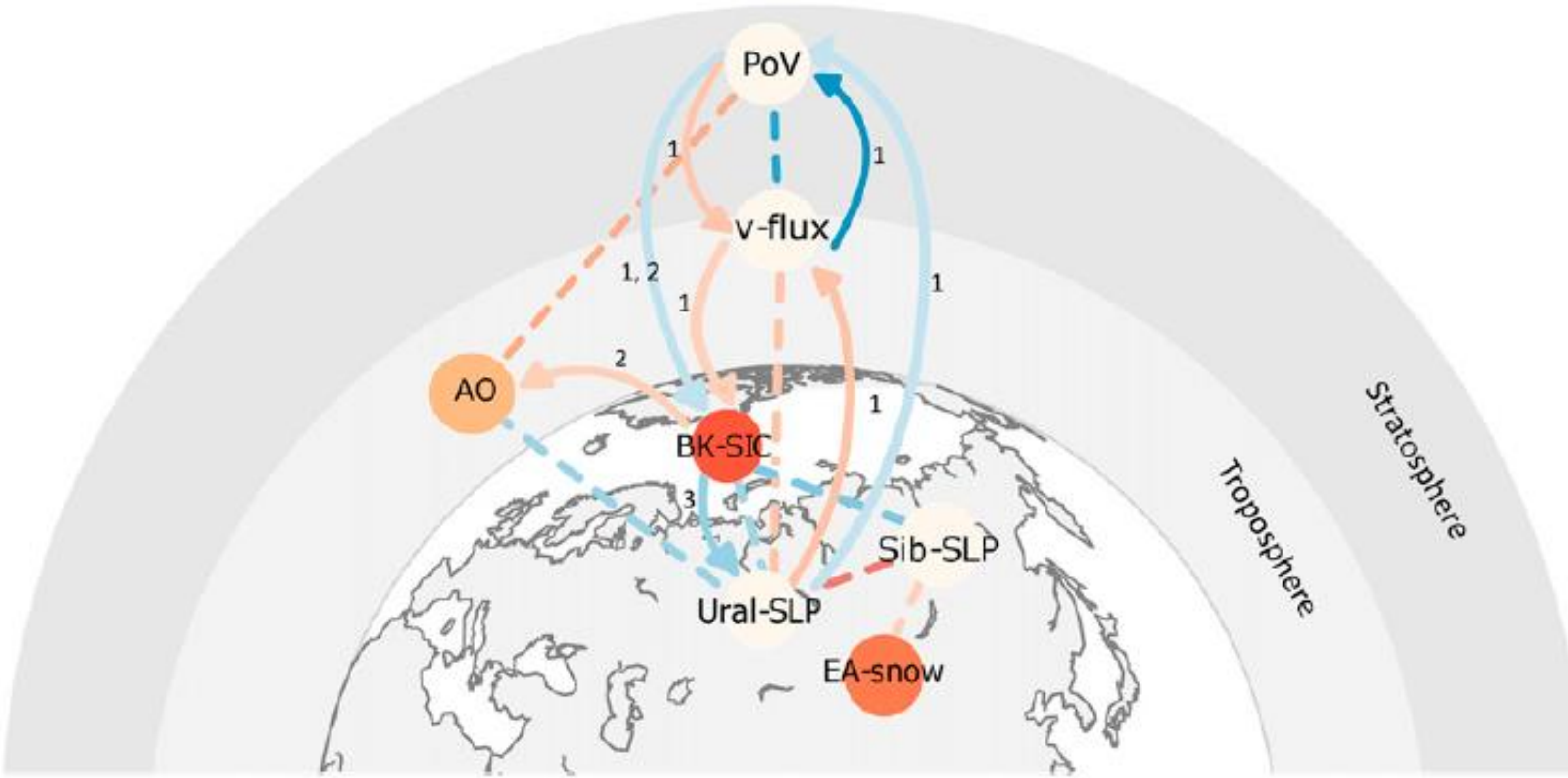
Runge et al., 2015

Example 1: Teleconnections between tropical Pacific and Indian oceans



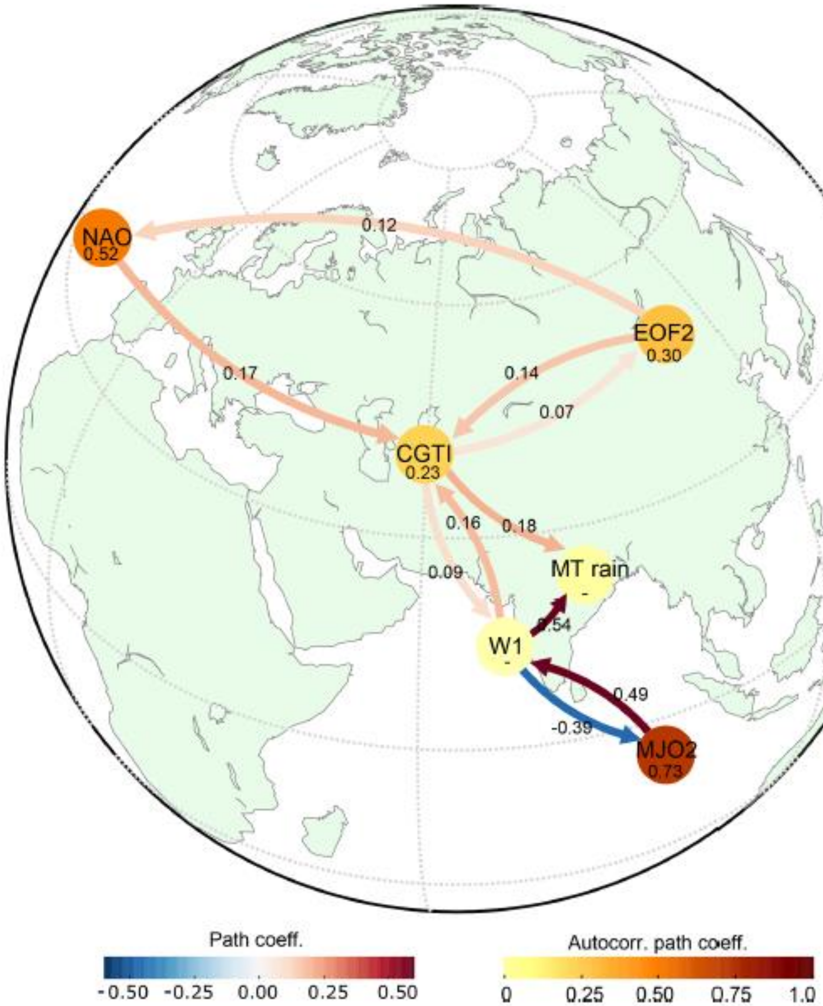
Runge et al., 2015

Example 2: Stratospheric polar vortex dynamics



Kretschmer et al., 2016

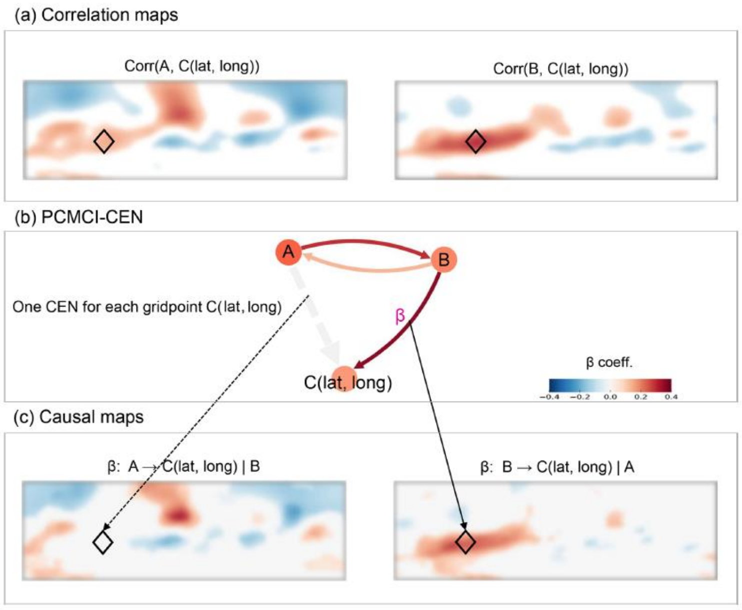
Example 3: Drivers of Indian Summer Monsoon

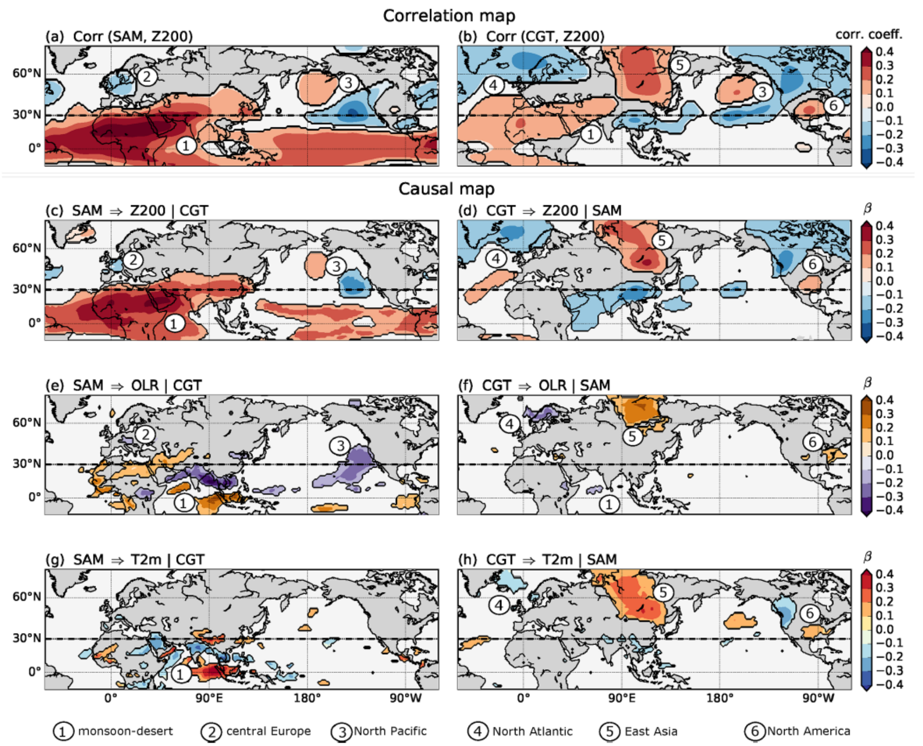


Di Capua et al., 2020

Causal maps

Spatial domain which is affected by a certain actor conditional on one or more others





Di Capua et al., 2020

Remaining challenges

- Predictive models from CEN

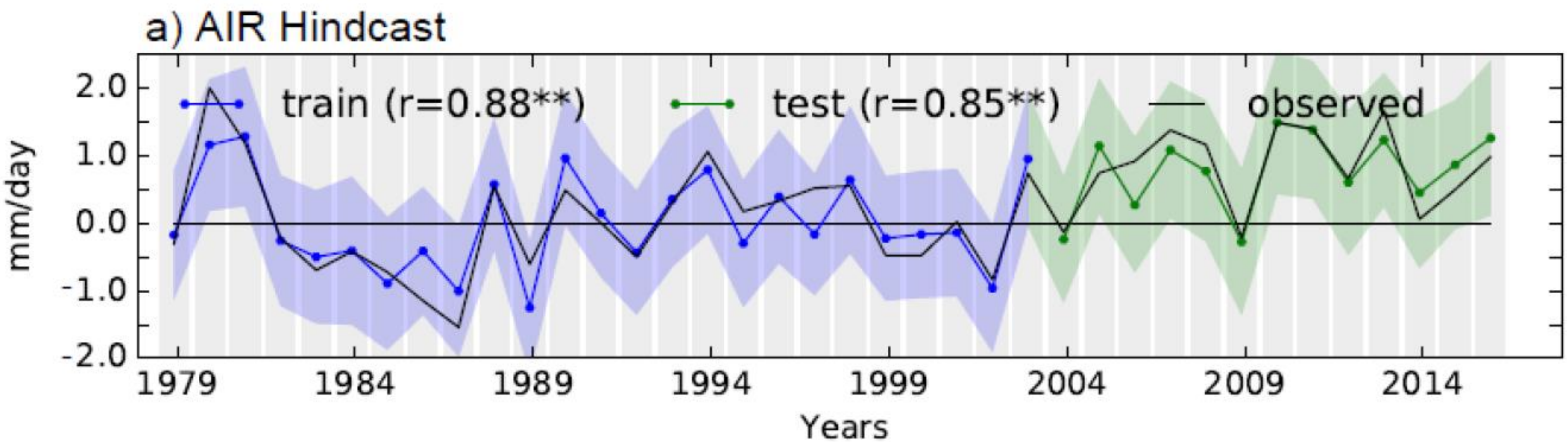
- Estimation of CEN from short time series with nonlinear dependency

- CEN analysis for variables changing on very different time scales
  - Dynamics at specific scales vs. dependencies across scales
  - May require different concepts of bivariate (conditional) dependency (e.g. phase-amplitude, phase-phase, amplitude-ampitude coupling, cf. Palus 2013)

- Time/state-dependent/non-stationary causal linkages
  - Exploration of processes leading to impactful extremes
  - Changes in climate dynamics under global warming

Predictive models from CEN

Straightforward if CEN based on partial correlations



Di Capua et al., 2019

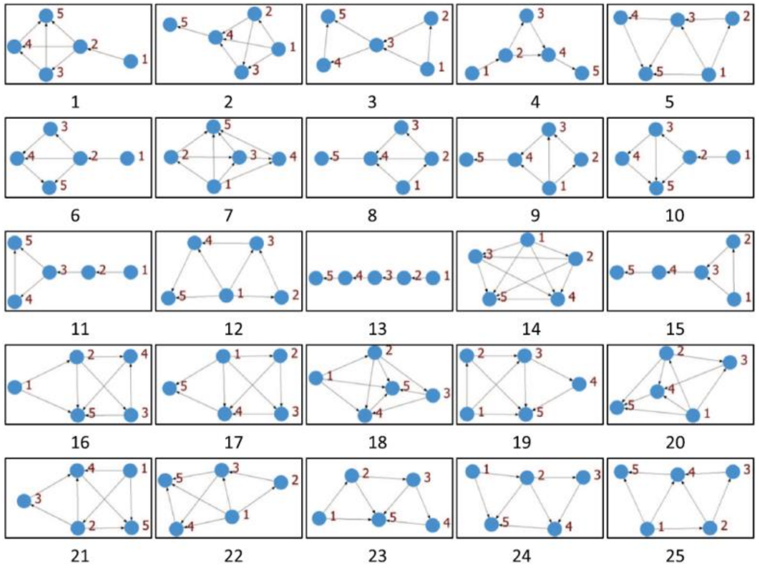
when being based on nonlinear similarity measure (e.g. conditional mutual information): training of a flexible regression model based on identified causal predictors (e.g. using neural network)

Possible alternative: learning ODE model from data (e.g. using SINDy approach)

How to escape the limited data problem?

Idea: replace sequences of continuous time series values (with embedding dimension *D* and embedding delay τ) by clever symbolization with an alphabet of low cardinality (few informative symbols)

- **Quantile classes: not very informative in case of slow variations**
- **Ordinal patterns: explicit amplitude information discarded, only relative order matters**
- **Visibility graphlets: visibility graphs among each sequence of length *D* (Mutua et al., Chaos, 2016; Wang et al., Chaos, 2019) – lower number of possible patterns than for ordinal encoding**



How to escape the limited data problem?

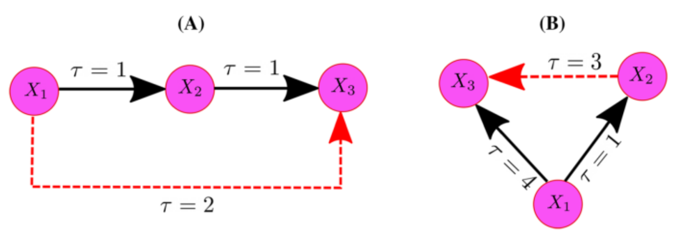
Numerical results for different model systems: pattern frequency based approaches may be less informative than pattern transition/co-occurrence frequency based approaches (Huang et al., Chaos, 2021)

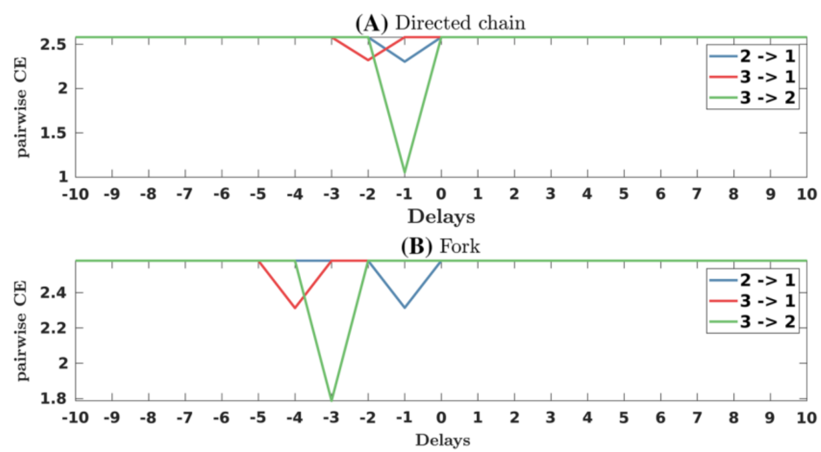
$\Rightarrow$ **Use (lagged) co-occurrence frequencies among all possible pairs of ordinal patterns**

Algorithm (Subramaniyam et al., Nonlin. Dyn., 2021):

1. **For each pair of time series, estimate co-occurrence frequencies of ordinal patterns at lag τ**
2. **Compute Shannon entropies of (conditional) co-occurrence frequencies = matrices of pairwise statistical associations indexed by considered time lag (Ruan et al., Chaos, 2019)**

$$H_\tau(X_2|X_1) = -\sum_{i=1}^{M!}\sum_{j=1}^{M!} p(\pi_i^{x_1,\tau}, \pi_j^{x_2}) \log_2 p(\pi_j^{x_2}|\pi_i^{x_1,\tau}),$$





Subramaniyam et al., 2021

How to escape the limited data problem?

Idea 2: pattern frequency based approaches may be less informative than pattern transition/co-occurrence frequency based approaches (Huang et al., Chaos, 2021) and do not inform on the bivariate case

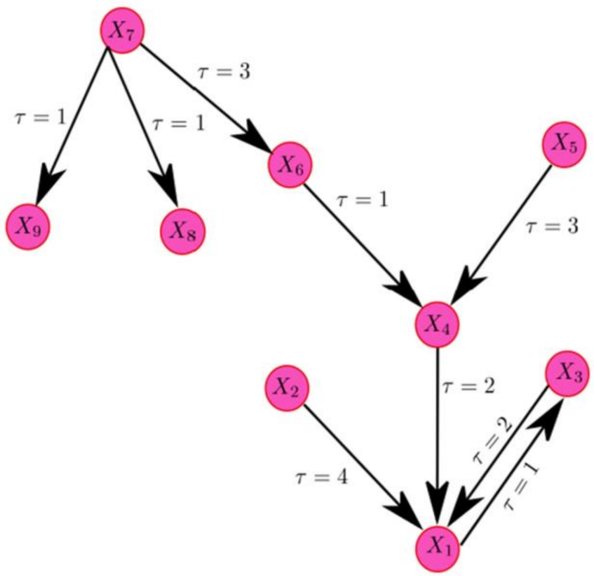
$\Rightarrow$ **Use (lagged) co-occurrence frequencies among all possible ordinal patterns**

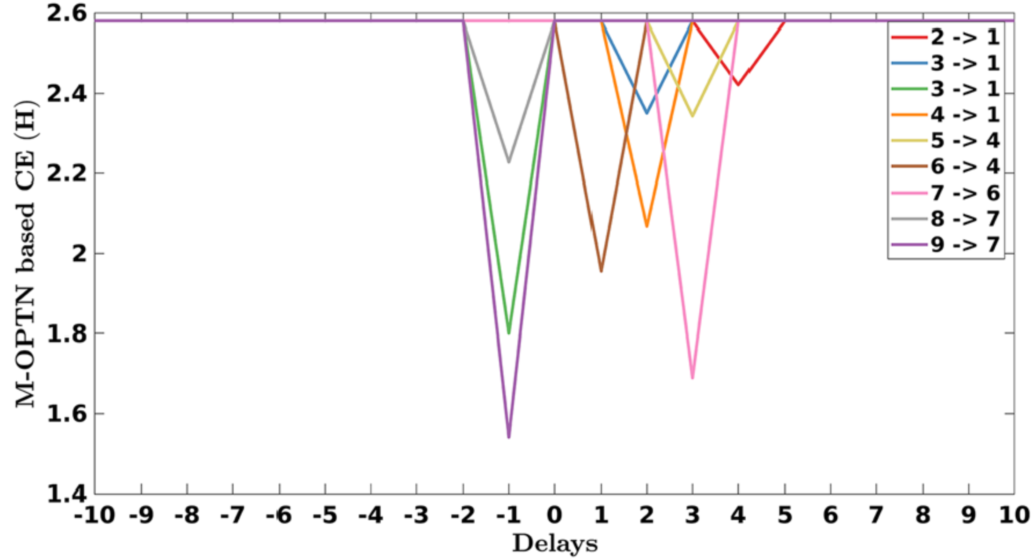
Algorithm (Subramaniyam et al., Nonlin. Dyn., 2021):

1. **For each pair of time series, estimate co-occurrence frequencies of ordinal patterns at lag τ**
2. **Compute Shannon entropies of (conditional) co-occurrence frequencies = matrices of pairwise statistical associations indexed by considered time lag (Ruan et al., Chaos, 2019)**
3. **Find sets of abundant parents and children for each pattern**
4. **Identify minimal conditioning sets**
5. **Remove non-causal interactions by proper conditioning**

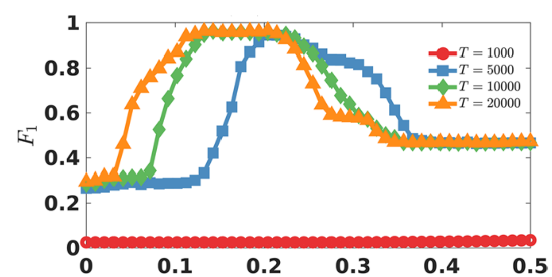
Cautionary notes: Process in steps 3-5 involves several algorithmic parameters for numerical stabilization!

Example: 9 coupled AR processes





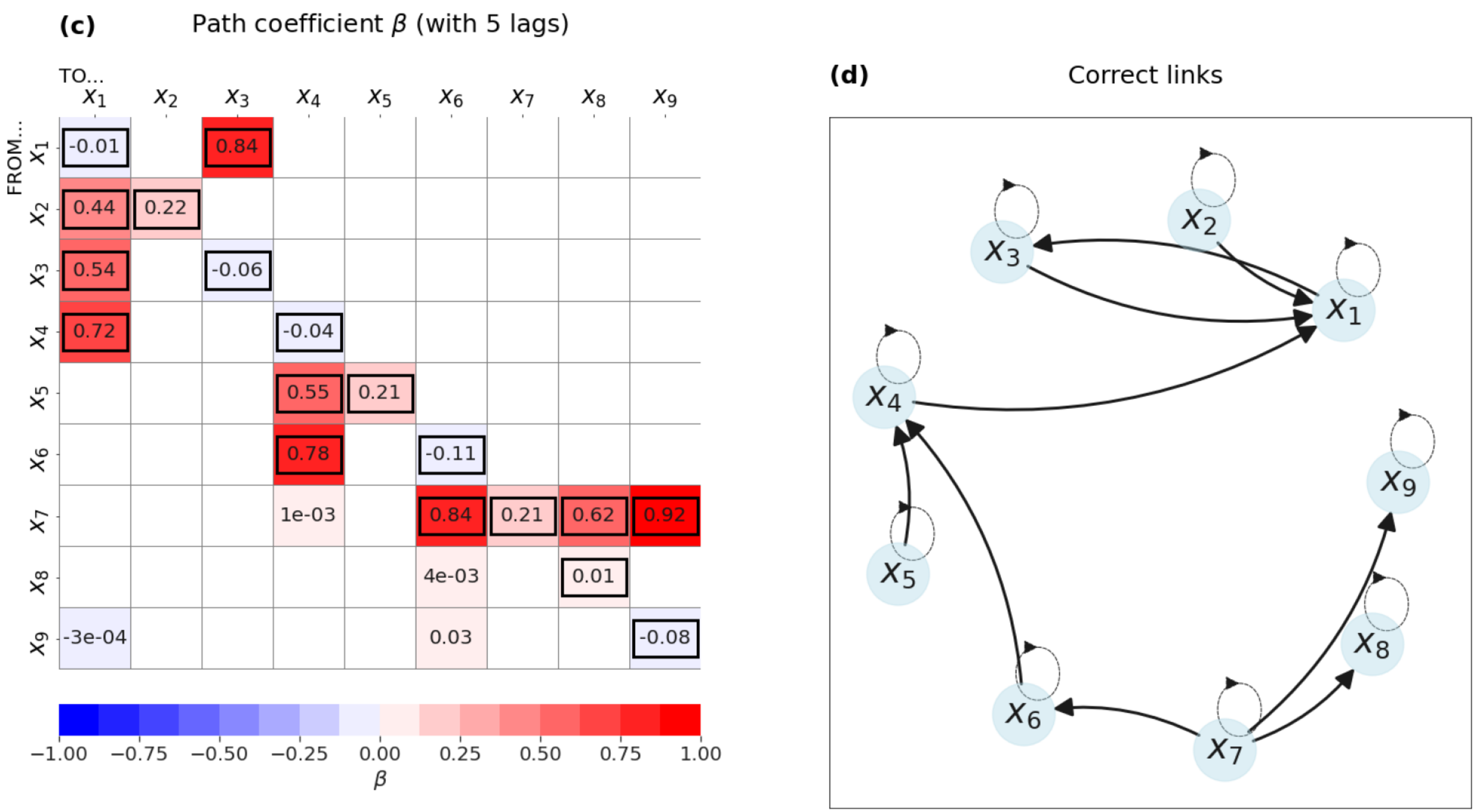
Robust correct results over a wide range of regularization parameters



Subramaniyam et al., 2021

Example: 9 coupled AR processes

Same problem addressed using PCMCI based on partial correlations



Docquier et al., in press

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Hochschule Magdeburg • Stendal

Take home messages

- **Causal effect network approach for identifying and quantifying causal relationships among a set of variables**

- **Successful applications to problems in space weather and climate variability, but also others (not shown)**

- **Specific algorithms available for special cases (e.g., instantaneous linkages, latent variables, etc.): Python package tigramite by Jakob Runge**

- **Ongoing work: method adaptations/applications to specific types of data (e.g. timing of events), circumventing limited data problems by clever choice of patterns**

Announcement: PhD position likely available from 1 April 2024 (only non-German EU citizens) – please contact me for details.