PADERBORN UNIVERSITY

Scaling Machine Learning on Knowledge Graphs

Keynote at EGC 2023

Axel Ngonga

EGC Lyon 2023

January 18, 2023

Introduction

Disclaimer

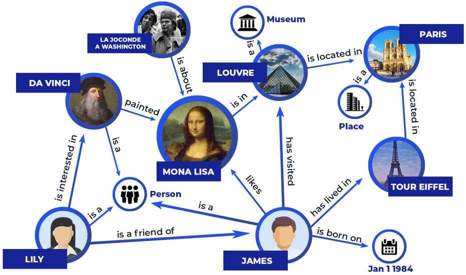
- Very incomplete
- Assumes familiarity with description logics

Section 1

Motivation

Motivation

Example



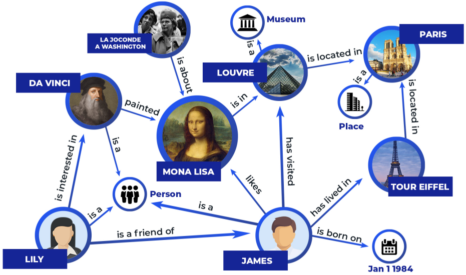
1

- $E^{+} = \{Louvre, TourEiffel\}$
- $E^{-} = \{Lily, James\}$

Source: https://bit.ly/3sxCj6e

Motivation

Example



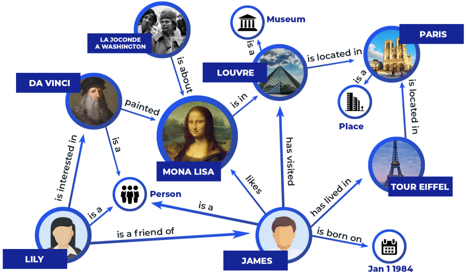
[1]

- $E^{+} = \{Louvre, TourEiffel\}$
- $E^{-} = \{Lily, James\}$
- $\mathcal{H} = \{\exists\, isLocatedIn.Place, \exists\, isLocatedIn.\{Paris\}\}$

Source: https://bit.ly/3sxCj6e

Motivation

Example



1

- $E^{+} = \{Louvre, TourEiffel\}$
- $E^{-} = \{Lily, James\}$
- $\mathcal{H} = \{\exists\, isLocatedIn.Place, \exists\, isLocatedIn.\{Paris\}\}$

Pros and Cons

- Pro: explainable, exploits background knowledge
- Contra: slow :-(

Source: https://bit.ly/3sxCj6e

Motivation

Let's play!


[a]

[a] https://www.flickr.com/photos/willwm/2065975725

Motivation

Let's play!

- What is 3+3?

 [a]

[a] https://www.flickr.com/photos/willwm/2065975725

Motivation

Let's play!

- What is 3+3?
- Square root of 4?

[a]

[a] https://www.flickr.com/photos/willwm/2065975725

Motivation

Let's play!

- What is 3+3?
- Square root of 4?
- What's the capital of France?

[a]

[a] https://www.flickr.com/photos/willwm/2065975725

Motivation

Let's play!

- What is 3+3?
- Square root of 4?
- What's the capital of France?
- Close your eyes.

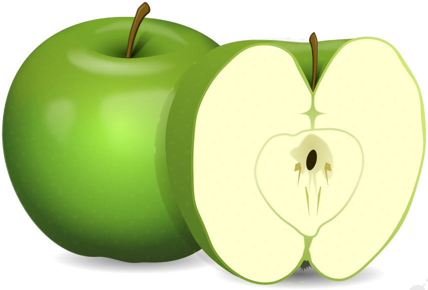
 [a]

[a] https://www.flickr.com/photos/willwm/2065975725

Motivation

How does the brain form thoughts?

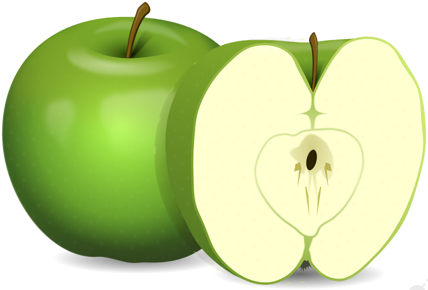
- System 1 [Kahneman, 2011]
  - Intuitive responses
  - Time-efficient
  - Unconscious



Motivation

How does the brain form thoughts?

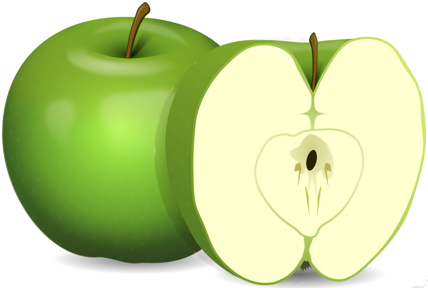
- System 1 [Kahneman, 2011]
  - Intuitive responses
  - Time-efficient
  - Unconscious
- System 2
  - Logical responses
  - Resource-intensive
  - Conscious



Motivation

How does the brain form thoughts?

- System 1 [Kahneman, 2011]
  - Intuitive responses
  - Time-efficient
  - Unconscious
- System 2
  - Logical responses
  - Resource-intensive
  - Conscious
- Both trainable and configurable



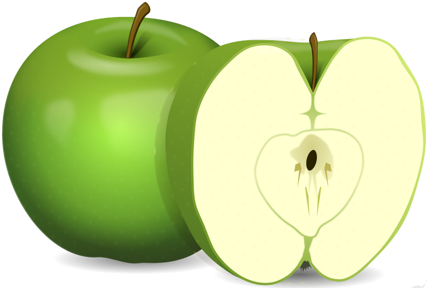
Motivation

How does the brain form thoughts?

In a nutshell

- Multiple representations seem to be beneficial for rapid cognition
- Can they help improve the runtime of class expression learning?

- System 1 [Kahneman, 2011]
  - Intuitive responses
  - Time-efficient
  - Unconscious
- System 2
  - Logical responses
  - Resource-intensive
  - Conscious
- Both trainable and configurable



Section 2

Class Expression Learning

Class Expression Learning

Formal definition

- Supervised learning with background knowledge (adapted from [Lehmann and Hitzler, 2010])
- Given:
  - Formal logic $\mathcal{L}$, e.g. $\mathcal{ALC}$
  - Background knowledge in form of knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$
  - Set of positive examples $E^+ \subseteq N_I$
  - Set of negative examples $E^- \subseteq N_I$

Class Expression Learning

Formal definition

- Supervised learning with background knowledge (adapted from [Lehmann and Hitzler, 2010])
- Given:
  - Formal logic $\mathcal{L}$, e.g. $\mathcal{ALC}$
  - Background knowledge in form of knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$
  - Set of positive examples $E^+ \subseteq N_I$
  - Set of negative examples $E^- \subseteq N_I$
- Goal: Find at least one hypothesis $H \in \mathcal{H}$ with
  1. H is a class expression in $\mathcal{L}$, and (ideally)
  2. $\forall e^+ \in E^+ : \mathcal{K} \models H(e^+)$
  3. $\forall e^- \in E^- : \mathcal{K} \not\models H(e^-)$

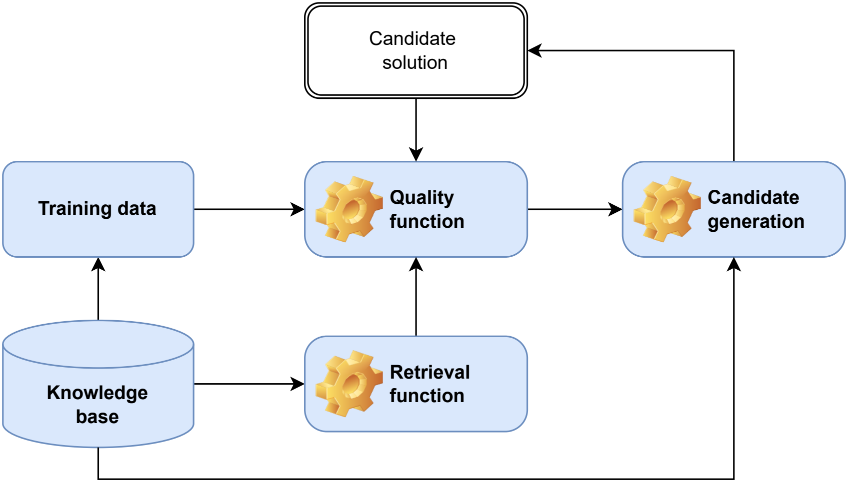
Class Expression Learning

Formal definition

- Supervised learning with background knowledge (adapted from [Lehmann and Hitzler, 2010])
- Given:
  - Formal logic $\mathcal{L}$, e.g. $\mathcal{ALC}$
  - Background knowledge in form of knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$
  - Set of positive examples $E^+ \subseteq N_I$
  - Set of negative examples $E^- \subseteq N_I$
- Goal: Find at least one hypothesis $H \in \mathcal{H}$ with
  1. H is a class expression in $\mathcal{L}$, and (ideally)
  2. $\forall e^+ \in E^+ : \mathcal{K} \models H(e^+)$
  3. $\forall e^- \in E^- : \mathcal{K} \not\models H(e^-)$
- Practically, aim to find $H \in \underset{C \in \mathcal{L}}{argmax}\ Q(C)$ [Heindorf et al., 2022]

Class Expression Learning

Common Approach



Class Expression Learning

Example: $\mathcal{L} = \mathcal{ALC}$

- Let C and D be $\mathcal{ALC}$ concepts
- Let $r \in N_R$ be a role
- Then, the following are $\mathcal{ALC}$ concepts [Schmidt-Schauß and Smolka, 1991]

Syntax	Semantics
$\top$	$\Delta^{\mathcal{I}}$
$\bot$	$\emptyset$
$C \in N_C$	$C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
$\exists r.C$	$\{x \in \Delta^{\mathcal{I}} : \exists y \in C^{\mathcal{I}} \text{ with } (x, y) \in r^{\mathcal{I}}\}$
$\forall r.C$	$\{x \in \Delta^{\mathcal{I}} : (x, y) \in r^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$

Class Expression Learning

Example: Refinement Operator

- Let $(S, \sqsubseteq)$ be a space with a quasi-ordering
- A top-down refinement operator $\rho : S \to 2^S$ is a mapping with $\rho(x) \sqsubseteq x$ [Lehmann and Hitzler, 2010]

Class Expression Learning

Example: Refinement Operator

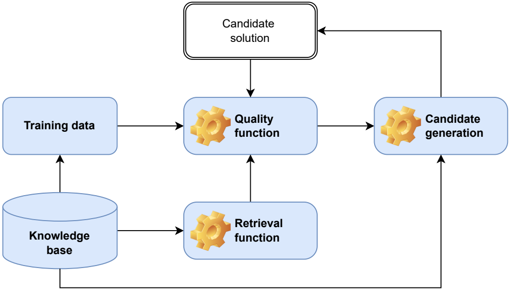
- Let $(S, \sqsubseteq)$ be a space with a quasi-ordering
- A top-down refinement operator $\rho : S \rightarrow 2^S$ is a mapping with $\rho(x) \sqsubseteq x$ [Lehmann and Hitzler, 2010]

Example

- Let S be the set of all concepts in our language $\mathcal{L} = \mathcal{EL}$
- The following operator ρ is a top-down refinement operator
- $\rho(C) = \begin{cases} C & \\ N_C \cup \{\exists r_j.\rho(C_i)\} & \text{if } C = \top \\ \rho(D) & \text{if } D \sqsubseteq C \\ C \sqcap D & \text{with } D \in N_C \\ C \sqcap \exists r.\rho(D) & \text{with } D \in N_C \end{cases}$

Learning problem

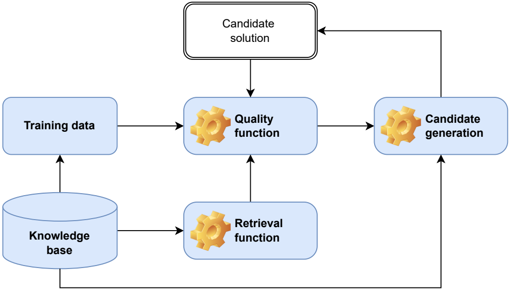
Challenges



- Retrieval is expensive

Learning problem

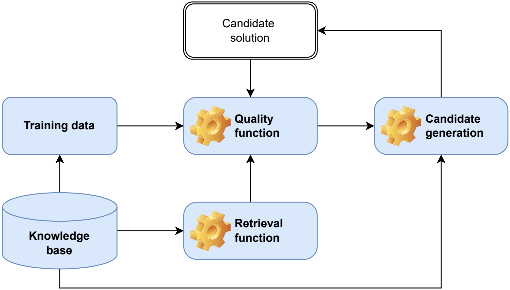
Challenges



- Retrieval is expensive $\Rightarrow$ Exploit SPARQL
- Quality functions are often myopic

Learning problem

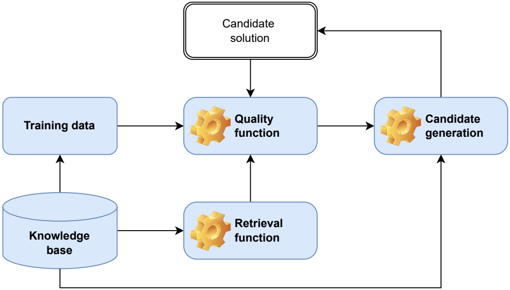
Challenges



- Retrieval is expensive ⇒ Exploit SPARQL
- Quality functions are often myopic ⇒ Exploit embeddings
- Candidate generation is expensive

Learning problem

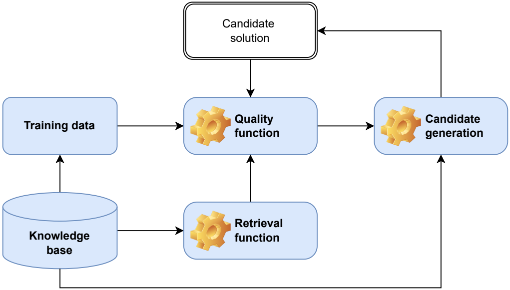
Challenges



- Retrieval is expensive $\Rightarrow$ Exploit SPARQL
- Quality functions are often myopic $\Rightarrow$ Exploit embeddings
- Candidate generation is expensive $\Rightarrow$ Exploit priming

Learning problem

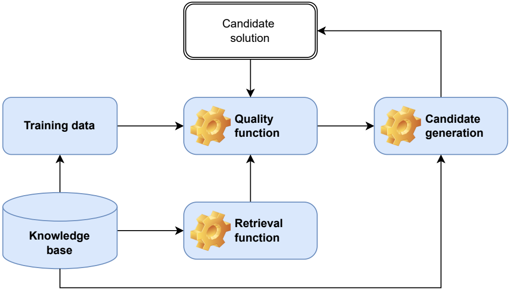
Challenges



- Retrieval is expensive $\Rightarrow$ Exploit SPARQL
- Quality functions are often myopic $\Rightarrow$ Exploit embeddings
- Candidate generation is expensive $\Rightarrow$ Exploit priming
- Search space is large

Learning problem

Challenges



- Retrieval is expensive ⇒ Exploit SPARQL
- Quality functions are often myopic ⇒ Exploit embeddings
- Candidate generation is expensive ⇒ Exploit priming
- Search space is large ⇒ Prune by length

Section 3

Representing Concepts as SPARQL

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

Class Expression	Graph Pattern $\mathfrak{p} = \tau(C_i, \texttt{?var})$
$A \in N_C$	`?var rdf:type A.`

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

Class Expression	Graph Pattern $\mathfrak{p} = \tau(C_i, \texttt{?var})$
$A \in N_C$	`?var rdf:type A.`
$\neg C$	`{?var ?p ?o} UNION {?s ?p ?var}.` FILTER NOT EXISTS $\{\tau(C, \texttt{?var})\}$

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

Class Expression	Graph Pattern $\mathfrak{p} = \tau(C_i, \texttt{?var})$
$A \in N_C$	`?var rdf:type A.`
$\neg C$	`{?var ?p ?o} UNION {?s ?p ?var}.` `FILTER NOT EXISTS` $\{\tau(C, \texttt{?var})\}$
$C_1 \sqcap \ldots \sqcap C_n$	$\{\tau(C_1, ?var) \ldots \tau(C_n, ?var)\}$

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

Class Expression	Graph Pattern $\mathfrak{p} = \tau(C_i, \texttt{?var})$
$A \in N_C$	`?var rdf:type A.`
$\neg C$	`{?var ?p ?o} UNION {?s ?p ?var}.` `FILTER NOT EXISTS` $\{\tau(C, \texttt{?var})\}$
$C_1 \sqcap \ldots \sqcap C_n$	$\{\tau(C_1, ?\text{var}) \ldots \tau(C_n, ?\text{var})\}$
$C_1 \sqcup \ldots \sqcup C_n$	$\{\tau(C_1, ?\text{var})\}$ `UNION` $\ldots$ `UNION` $\{\tau(C_n, ?\text{var})\}$

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

Class Expression	Graph Pattern $\mathfrak{p} = \tau(C_i, \texttt{?var})$
$A \in N_C$	`?var rdf:type A.`
$\neg C$	`{?var ?p ?o} UNION {?s ?p ?var}.` `FILTER NOT EXISTS` $\{\tau(C, \texttt{?var})\}$
$C_1 \sqcap \ldots \sqcap C_n$	$\{\tau(C_1, ?\text{var}) \ldots \tau(C_n, ?\text{var})\}$
$C_1 \sqcup \ldots \sqcup C_n$	$\{\tau(C_1, ?\text{var})\}$ `UNION` ... `UNION` $\{\tau(C_n, ?\text{var})\}$
$\exists\, r.C$	`{?var r ?s.` $\tau(C, ?\text{s})\}$

Representing Concepts as SPARQL

From $\mathcal{ALC}$ to SPARQL

- Assume closed world and fully materialized knowledge graph
- Retrieval in $\mathcal{ALC}$ can be realized by representing concepts as SPARQL queries [Bin et al., 2016]

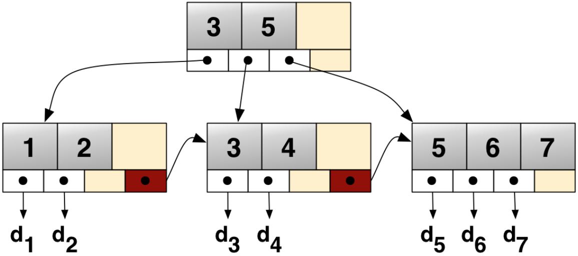
Class Expression	Graph Pattern $\mathfrak{p} = \tau(C_i, \texttt{?var})$
$A \in N_C$	`?var rdf:type A.`
$\neg C$	`{?var ?p ?o} UNION {?s ?p ?var}.` `FILTER NOT EXISTS` $\{\tau(C, \texttt{?var})\}$
$C_1 \sqcap \ldots \sqcap C_n$	$\{\tau(C_1, ?var) \ldots \tau(C_n, ?var)\}$
$C_1 \sqcup \ldots \sqcup C_n$	$\{\tau(C_1, ?var)\}$ `UNION` $\ldots$ `UNION` $\{\tau(C_n, ?var)\}$
$\exists\, r.C$	`{?var r ?s.` $\tau(C, ?s)\}$

$\forall\, r.C$	`{ ?var r ?s0.`
	`{ SELECT ?var (count(?s1) AS ?cnt1)`
	`WHERE { ?var r ?s1.` $\tau(C,$ `?s1)}`
	`GROUP BY ?var }`
	`{ SELECT ?var (count(?s2) AS ?cnt2)`
	`WHERE { ?var r ?s2 .}`
	`GROUP BY ?var }`
	`FILTER ( ?cnt1 = ?cnt2 ) }`

Representing Concepts as SPARQL

Storage Solutions

- Important difference are indexing data structures
- Typical indexes include
  - Resource index, e.g., a hash table
  - Triple index, e.g., a B^+ tree

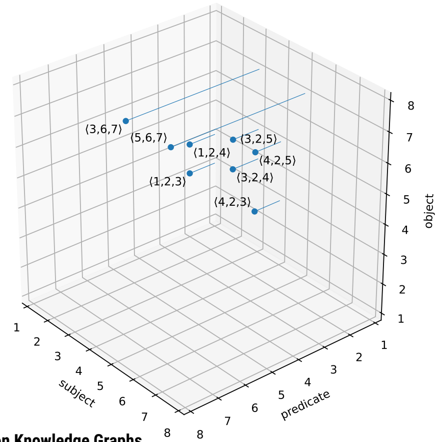


Representing Concepts as SPARQL

Tentris: Idea

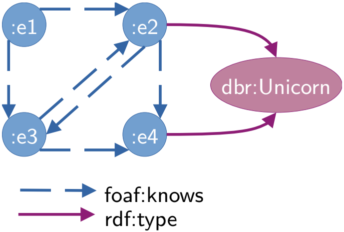
Idea [Bigerl et al., 2020]

- Exploit tensor representation to accelerate querying
- Devise data structure to accommodate rapid querying



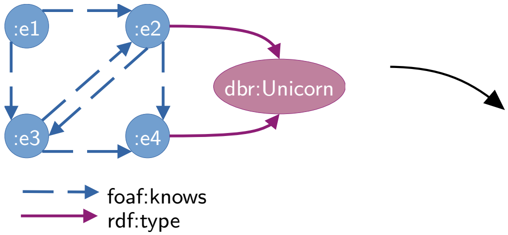
Representing Concepts as SPARQL

From RDF to Tensors



Representing Concepts as SPARQL

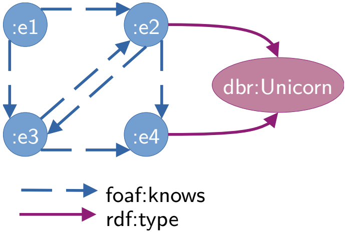
From RDF to Tensors



term	*id*(term)
:e1	1
foaf:knows	2
:e2	3
:e3	4
:e4	5
rdf:type	6
dbr:Unicorn	7
unbound	8

Representing Concepts as SPARQL

From RDF to Tensors

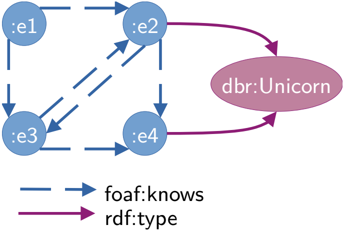


term	*id*(term)
:e1	1
foaf:knows	2
:e2	3
:e3	4
:e4	5
rdf:type	6
dbr:Unicorn	7
unbound	8

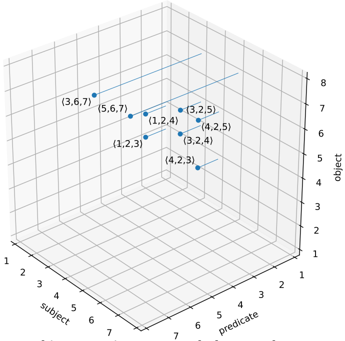
id(s)	*id*(p)	*id*(o)
1	2	3
1	2	4
3	2	4
3	2	5
4	2	3
4	2	5
3	6	7
5	6	7

Representing Concepts as SPARQL

From RDF to Tensors



term	*id*(term)
:e1	1
foaf:knows	2
:e2	3
:e3	4
:e4	5
rdf:type	6
dbr:Unicorn	7
unbound	8



Representing Concepts as SPARQL

TENTRIS: Data Model

- Consider order-n tensors $T : \mathbf{K} = \mathbf{K}_1 \times \cdots \times \mathbf{K}_n \to V$

Representing Concepts as SPARQL

TENTRIS: Data Model

- Consider order-n tensors $T : \mathbf{K} = \mathbf{K}_1 \times \cdots \times \mathbf{K}_n \rightarrow V$
  - $\mathbf{K}_1 = \cdots = \mathbf{K}_n \subset \mathbb{N}$

Representing Concepts as SPARQL

TENTRIS: Data Model

- Consider order-n tensors $T : \mathbf{K} = \mathbf{K}_1 \times \cdots \times \mathbf{K}_n \to V$
  - $\mathbf{K}_1 = \cdots = \mathbf{K}_n \subset \mathbb{N}$
  - $\mathbb{B}$ or $\mathbb{N}$ as co-domain

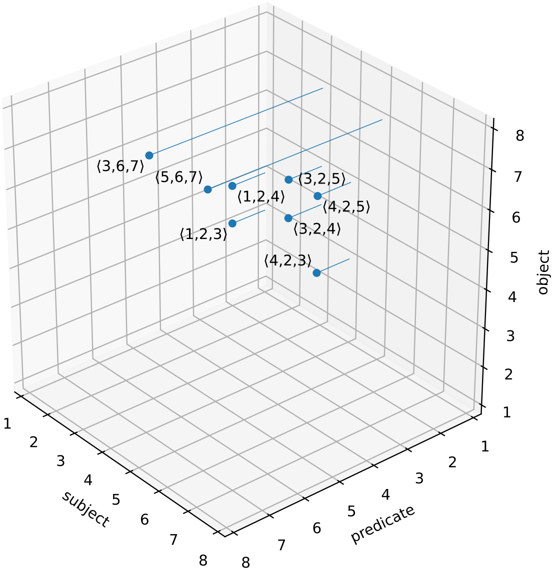
Representing Concepts as SPARQL

Tentris: Data Model

- Consider order-n tensors $T : \mathbf{K} = \mathbf{K}_1 \times \cdots \times \mathbf{K}_n \rightarrow V$
  - $\mathbf{K}_1 = \cdots = \mathbf{K}_n \subset \mathbb{N}$
  - $\mathbb{B}$ or $\mathbb{N}$ as co-domain
- $\mathbf{k} \in \mathbf{K}$ is a key with key parts $\langle \mathbf{k}_1, \ldots, \mathbf{k}_n \rangle$
- Values v in a tensor are accessed in array style, e.g., $T[\mathbf{k}] = v$

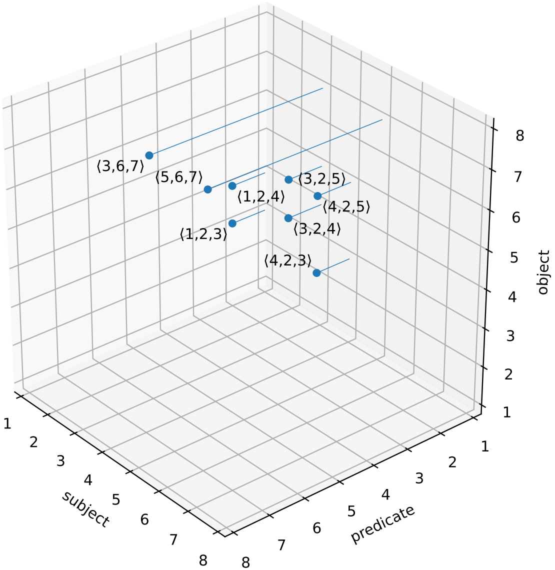
Representing Concepts as SPARQL

TENTRIS: Data Model



Representing Concepts as SPARQL

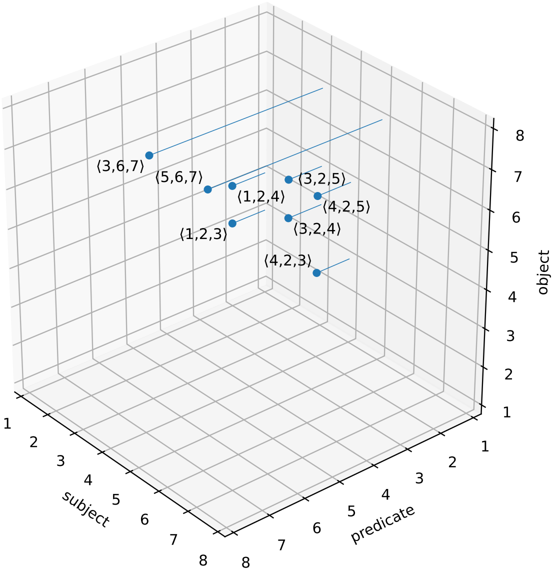
Tentris: Data Model



- $\mathbf{K} = \mathbb{N}^3$
- $V = \mathbb{B}$

Representing Concepts as SPARQL

TENTRIS: Data Model

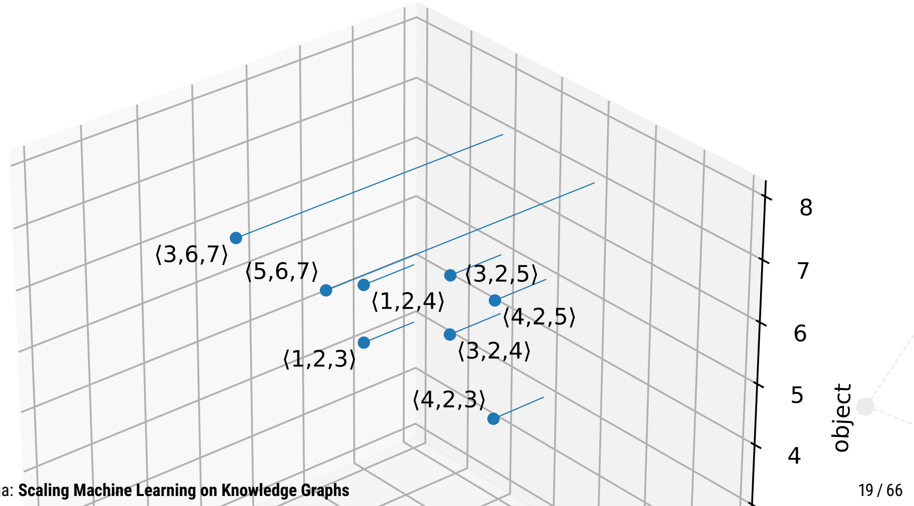


- $\mathbf{K} = \mathbb{N}^3$
- $V = \mathbb{B}$
- $T[\langle 3, 6, 7\rangle] = 1$
- $T[\langle 3, 6, 3\rangle] = 0$

Representing Concepts as SPARQL

Tentris: Data Model

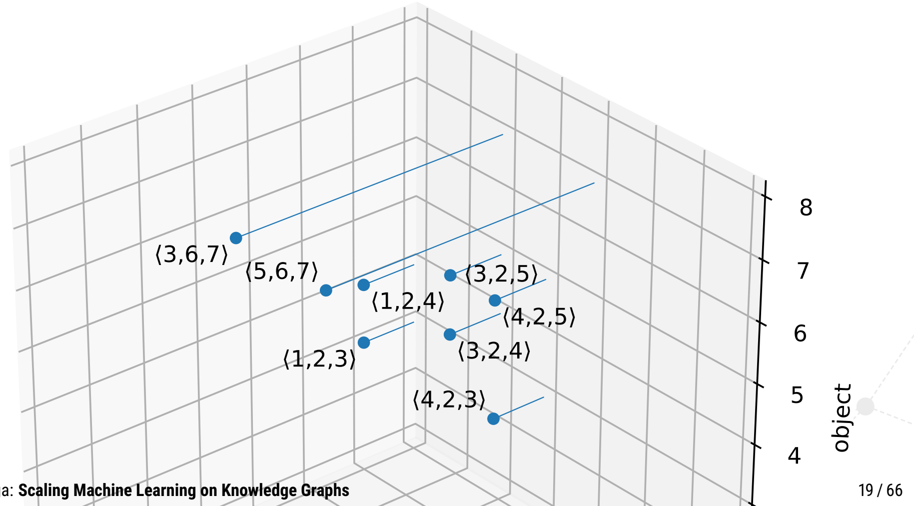
- Slicing selects portion of T, e.g., $T^{(1)} := T[1, 2, :]$ is order-1 tensor



Representing Concepts as SPARQL

TENTRIS: Data Model

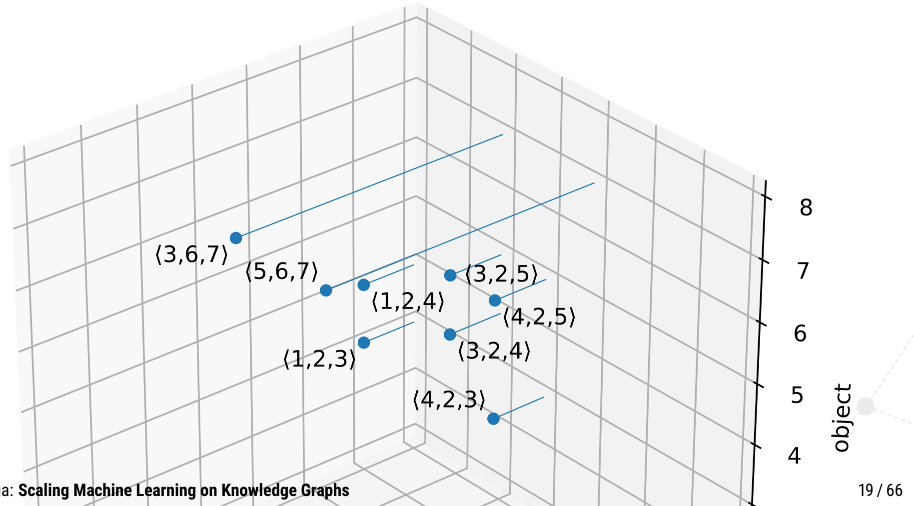
- Slicing selects portion of T, e.g., $T^{(1)} := T[1, 2, :]$ is order-1 tensor
- For our example, $T[1, 2, :] = [0, 0, 1, 1, 0, 0, 0, 0]$



Representing Concepts as SPARQL

TENTRIS: Data Model

- Slicing selects portion of T, e.g., $T^{(1)} := T[1, 2, :]$ is order-1 tensor
- For our example, $T[1, 2, :] = [0, 0, 1, 1, 0, 0, 0, 0]$
- Slices can be joined via Einstein summation [Barr, 1989]



Representing Concepts as SPARQL

TENTRIS–Einstein Summation

```
SELECT ?f WHERE {
  :e1 foaf:knows ?f .
  ?f  foaf:knows ?u .
  ?u  rdf:type dbr:Unicorn
}
```

Representing Concepts as SPARQL

Tentris–Einstein Summation

```
SELECT ?f WHERE {
    :e1 foaf:knows ?f .
    ?f  foaf:knows ?u .
    ?u  rdf:type dbr:Unicorn
}
```

$T[1, 2, :]$ $T[:, 2, :]$ $T[:, 6, 7]$

Representing Concepts as SPARQL

TENTRIS–Einstein Summation

```
SELECT ?f WHERE {
  :e1 foaf:knows ?f .
  ?f  foaf:knows ?u .
  ?u  rdf:type dbr:Unicorn
}
```

$T[1, 2, :]$ $\quad$ $T[:, 2, :]$ $\quad$ $T[:, 6, 7]$

$$R_f \leftarrow T[1, 2, :]_f \times T[:, 2, :]_{f,u} \times T[:, 6, 7]_u$$

Representing Concepts as SPARQL

Tentris: Querying

- Triple pattern is mapped to

$$\mathbf{k}_i^{(Q)} := \begin{cases} :, & \text{if } Q_i \in U, \\ id(Q_i), & \text{otherwise.} \end{cases}$$

Representing Concepts as SPARQL

Tentris: Querying

- Triple pattern is mapped to

$$\mathbf{k}_i^{(Q)} := \begin{cases} :, & \text{if } Q_i \in U, \\ id(Q_i), & \text{otherwise.} \end{cases}$$

- BGP $B = \{B^{(1)}, \ldots, B^{(r)}\}$ is given by

$$T'_{\langle l \in U \rangle} \leftarrow \bigtimes_i T[\mathbf{k}^{B^{(i)}}]_{\langle l \in B^{(i)} | l \in U \rangle}$$

Representing Concepts as SPARQL

TENTRIS: Querying

- Triple pattern is mapped to

$$\mathbf{k}_i^{(Q)} := \begin{cases} :, & \text{if } Q_i \in U, \\ id(Q_i), & \text{otherwise.} \end{cases}$$

- BGP $B = \{B^{(1)}, \ldots, B^{(r)}\}$ is given by

$$T'_{\langle l \in U \rangle} \leftarrow \bigtimes_i T[\mathbf{k}^{B^{(i)}}]_{\langle l \in B^{(i)} | l \in U \rangle}$$

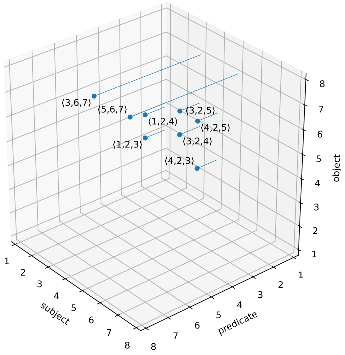
- The projection $\Pi_{U'}(B(g))$ with $U' \subseteq U$ is given by

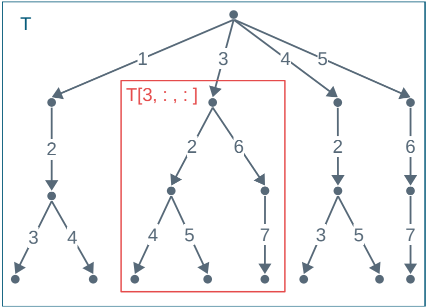
$$T''_{\langle l \in U' \rangle} \leftarrow \bigtimes_i T[\mathbf{k}^{B^{(i)}}]_{\langle l \in B^{(i)} | l \in U \rangle}$$

Representing Concepts as SPARQL

TENTRIS: Hypertrie

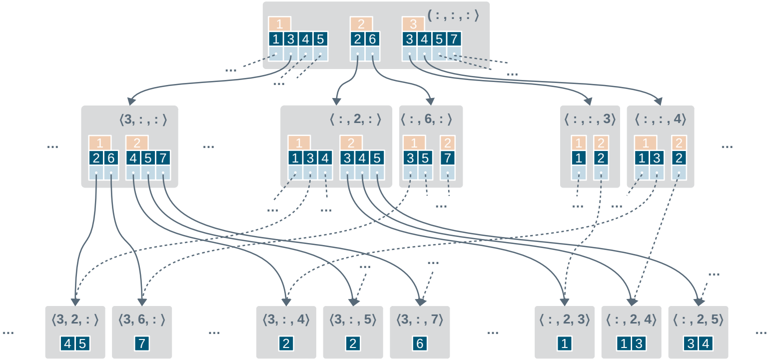
- Query for any tensor slice efficiently
- Allow for efficient querying





Representing Concepts as SPARQL

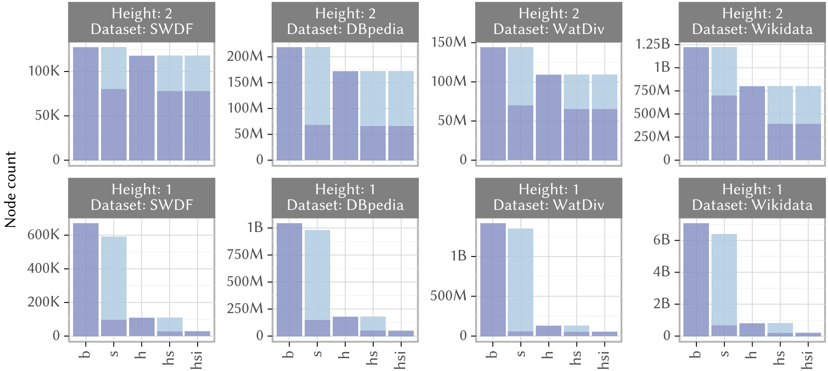
TENTRIS: Hypertrie



- Query for any tensor slice efficiently
- Storage bound is reduced from $\mathcal{O}(d! \cdot d \cdot z(h))$ for all collation orders to $\mathcal{O}(2^{d-1} \cdot d \cdot z(h))$

Representing Concepts as SPARQL

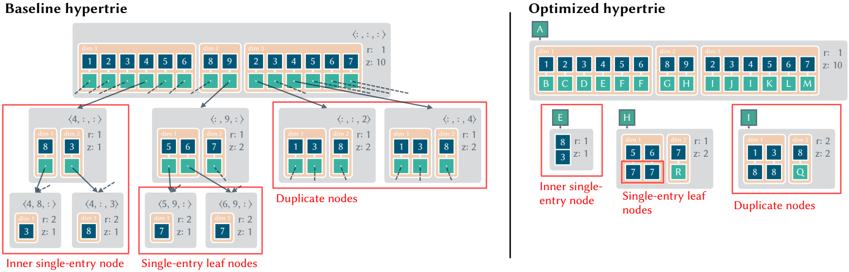
TENTRIS: Hypertrie



- Hypertrie topology seems sparse
- Compression to improve space, loading and query times [Bigerl et al., 2022]

Representing Concepts as SPARQL

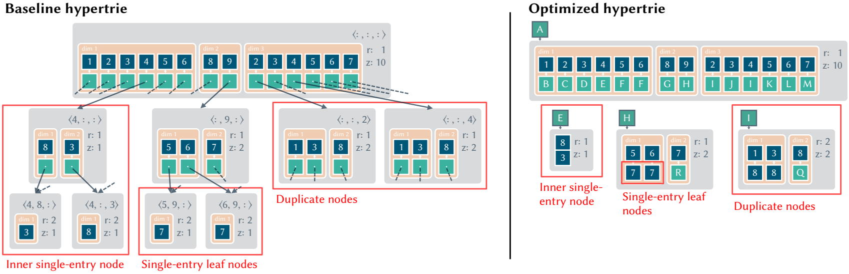
TENTRIS: Compressed Hypertrie



- Compress data based on local and global node topology

Representing Concepts as SPARQL

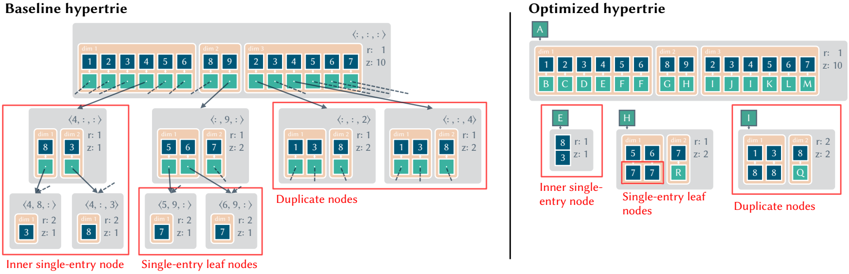
Tentris: Compressed Hypertrie



- Compress data based on local and global node topology
- 3 compression approaches
  1. Remove duplicates via hashing (global)

Representing Concepts as SPARQL

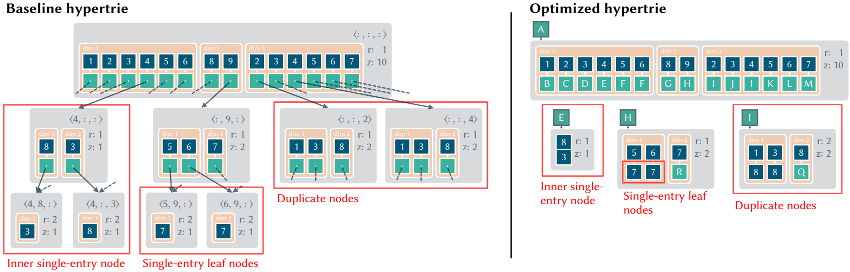
TENTRIS: Compressed Hypertrie



- Compress data based on local and global node topology
- 3 compression approaches
  1. Remove duplicates via hashing (global)
  2. Single-entry inner nodes (local) store sub-hypertries directly

Representing Concepts as SPARQL

TENTRIS: Compressed Hypertrie



- Compress data based on local and global node topology
- 3 compression approaches
  1. Remove duplicates via hashing (global)
  2. Single-entry inner nodes (local) store sub-hypertries directly
  3. Single-entry leaf nodes are eliminated via in-place storage (local)

Representing Concepts as SPARQL

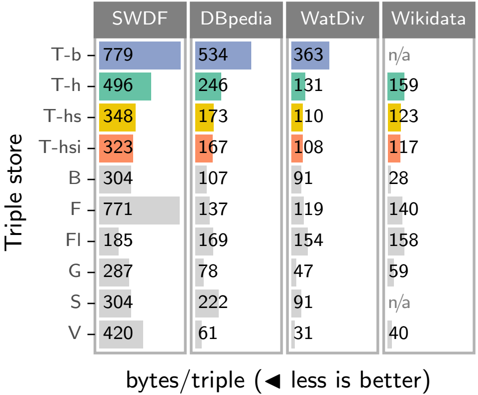
TENTRIS: Compressed Hypertrie

- Comparison with state-of-the-art approaches
- Hardware: AMD EPYC 7742, 1 TB RAM and 2×3 TB NVMe SSDs
- Datasets: Between 372K (SWDF) and 5.5B triples (WikiData)

Representing Concepts as SPARQL

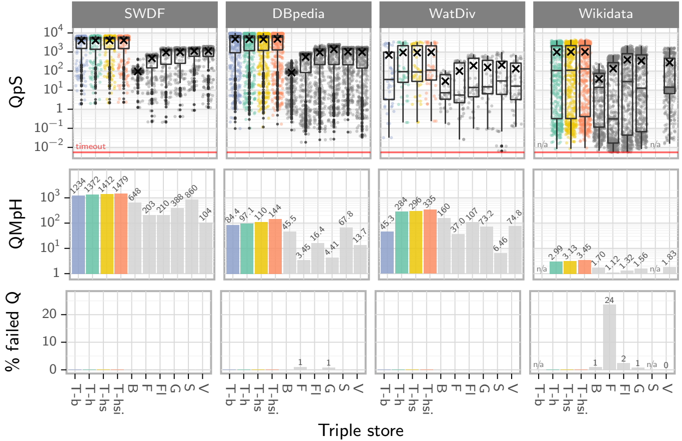
TENTRIS: Compressed Hypertrie

- Comparison with state-of-the-art approaches
- Hardware: AMD EPYC 7742, 1 TB RAM and 2×3 TB NVMe SSDs
- Datasets: Between 372K (SWDF) and 5.5B triples (WikiData)



Representing Concepts as SPARQL

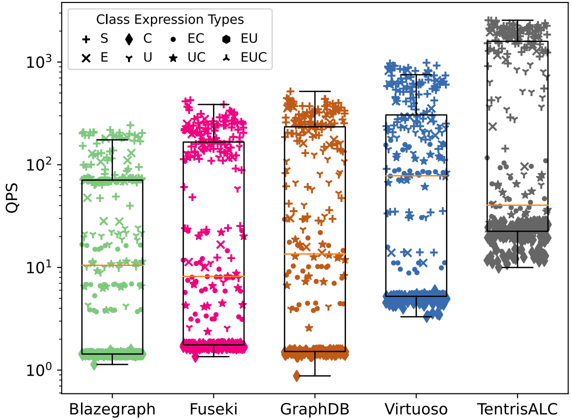
TENTRIS: Compressed Hypertrie



- Better runtimes on all datasets
- Can operate on very large datasets (no time-outs)

Representing Concepts as SPARQL

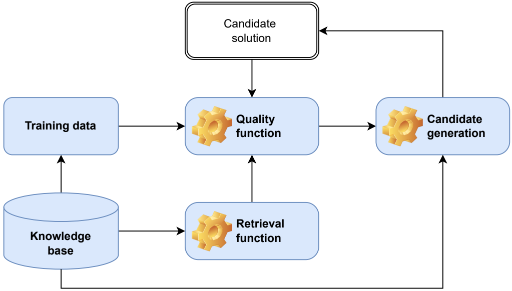
TENTRIS: Carcinogenesis



- Comparison on supervised machine learning tasks in $\mathcal{ALC}$
- Better runtimes on all datasets considered

Learning problem

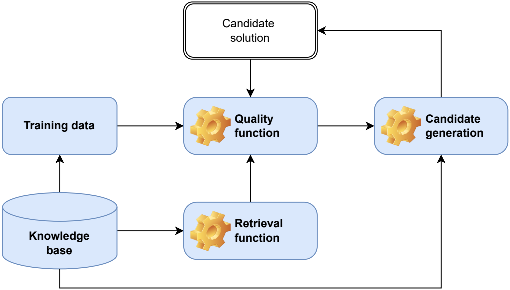
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ▶ Quality functions are often myopic

Learning problem

Challenges



- ✓ Retrieval is expensive $\Rightarrow$ Exploit SPARQL
- ► Quality functions are often myopic $\Rightarrow$ Exploit embeddings
- ► Candidate generation is expensive $\Rightarrow$ Exploit priming
- ► Search space is large $\Rightarrow$ Prune by length

Section 4

Improving Quality Functions

Improving Quality Functions

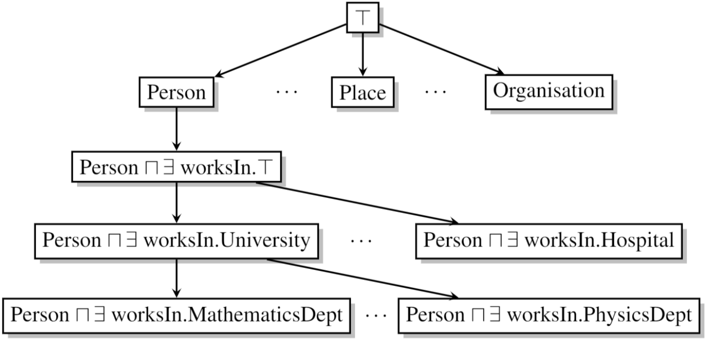
Refinement Operators

- Implement informed search in space $\mathcal{S}$ of all concepts with partial ordering $\sqsubseteq$
- Refinement operator $\rho : \mathcal{S} \rightarrow 2^{\mathcal{S}}$ with
  - $\forall x \in \rho(s) : x \sqsubseteq s$ (downward)
  - $\forall x \in \rho(s) : s \sqsubseteq x$ (upward)

Improving Quality Functions

Refinement Operators

- Implement informed search in space $\mathcal{S}$ of all concepts with partial ordering $\sqsubseteq$
- Refinement operator $\rho : \mathcal{S} \to 2^{\mathcal{S}}$ with
  - $\forall x \in \rho(s) : x \sqsubseteq s$ (downward)
  - $\forall x \in \rho(s) : s \sqsubseteq x$ (upward)



Improving Quality Functions

Quality Functions – OCEL

- Let $R(C)$ be the set of instances of C
- Let C' be the parent concept of C in the search tree

Improving Quality Functions

Quality Functions – OCEL

- Let $R(C)$ be the set of instances of C
- Let C' be the parent concept of C in the search tree
- Accuracy and accuracy gain of a concept C are defined as

$$\mathrm{acc}(C) = 1 - \frac{|E^+ \setminus R(C)| + |R(C) \cap E^-|}{|E|}$$

$$\mathrm{acc\_gain}(C) = \mathrm{acc}(C) - \mathrm{acc}(C')$$

Improving Quality Functions

Quality Functions – OCEL

- Let $R(C)$ be the set of instances of C
- Let C' be the parent concept of C in the search tree
- Accuracy and accuracy gain of a concept C are defined as

$$\mathrm{acc}(C) = 1 - \frac{|E^+ \setminus R(C)| + |R(C) \cap E^-|}{|E|}$$

$$\mathrm{acc\_gain}(C) = \mathrm{acc}(C) - \mathrm{acc}(C')$$

- The score is given by

$$\mathrm{score}(C) = \mathrm{acc}(C) + \alpha \cdot \mathrm{acc\_gain}(C) - \beta \cdot |C| \quad (\alpha, \beta \geq 0),$$

where $\alpha = 0.5$ and $\beta = 0.02$ are typical default values.

Improving Quality Functions

Quality Functions – CELOE

- Accuracy metric acc_c for CELOE:

$$\mathrm{acc}_c(C,t) = \frac{1}{t+1} \cdot \left(t \cdot \frac{|E^+ \cap R(C)|}{|E^+|} + \sqrt{\frac{|E^+ \cap R(C)|}{|R(C)|}} \right)$$

$$\mathrm{acc\_gain}_c(C) = \mathrm{acc}_c(C,t) - \mathrm{acc}_c(C',t)$$

Improving Quality Functions

Quality Functions – CELOE

- Accuracy metric acc_c for CELOE:

$$\mathrm{acc}_c(C,t) = \frac{1}{t+1} \cdot \left(t \cdot \frac{|E^+ \cap R(C)|}{|E^+|} + \sqrt{\frac{|E^+ \cap R(C)|}{|R(C)|}} \right)$$

$$\mathrm{acc\_gain}_c(C) = \mathrm{acc}_c(C,t) - \mathrm{acc}_c(C',t)$$

- $\mathrm{score}(C) = \mathrm{acc}_c(C,t) + \alpha \cdot \mathrm{acc\_gain}_c(C) - \beta \cdot |C| \quad (\alpha, \beta \geq 0)$
  where typical values are $\alpha = 0.3$ and $\beta = 0.05$.

Improving Quality Functions

Quality Functions – CELOE

- Accuracy metric acc_c for CELOE:

$$\mathrm{acc}_c(C,t) = \frac{1}{t+1} \cdot \left(t \cdot \frac{|E^+ \cap R(C)|}{|E^+|} + \sqrt{\frac{|E^+ \cap R(C)|}{|R(C)|}} \right)$$

$$\mathrm{acc\_gain}_c(C) = \mathrm{acc}_c(C,t) - \mathrm{acc}_c(C',t)$$

- $\mathrm{score}(C) = \mathrm{acc}_c(C,t) + \alpha \cdot \mathrm{acc\_gain}_c(C) - \beta \cdot |C| \quad (\alpha, \beta \geq 0)$
where typical values are $\alpha = 0.3$ and $\beta = 0.05$.

Problem: Myopia

- Current metrics do not consider future accuracy of concepts

Quality Functions – CELOE

- Accuracy metric acc_c for CELOE:

$$\mathrm{acc}_c(C,t) = \frac{1}{t+1} \cdot \left(t \cdot \frac{|E^+ \cap R(C)|}{|E^+|} + \sqrt{\frac{|E^+ \cap R(C)|}{|R(C)|}} \right)$$

$$\mathrm{acc\_gain}_c(C) = \mathrm{acc}_c(C,t) - \mathrm{acc}_c(C',t)$$

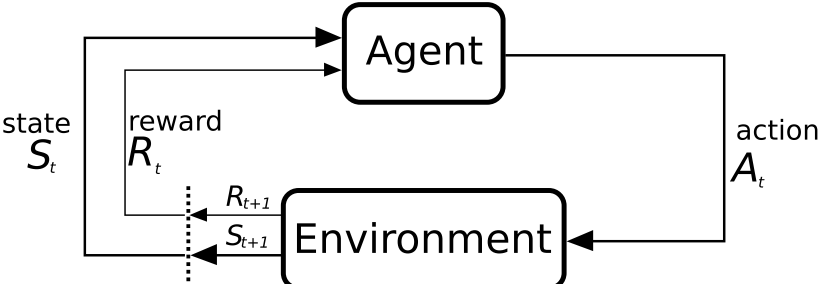
- $\mathrm{score}(C) = \mathrm{acc}_c(C,t) + \alpha \cdot \mathrm{acc\_gain}_c(C) - \beta \cdot |C| \quad (\alpha, \beta \geq 0)$
  where typical values are $\alpha = 0.3$ and $\beta = 0.05$.

Problem: Myopia

- Current metrics do not consider future accuracy of concepts
- Optimize for cumulative discounted future rewards
  [Demir and Ngonga Ngomo, 2021]

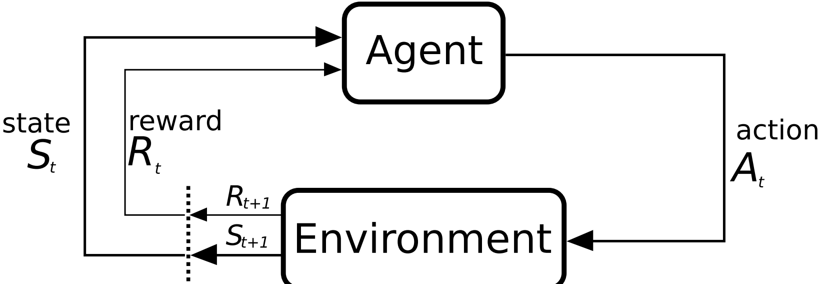
Improving Quality Functions

Reinforcement Learning



Improving Quality Functions

Reinforcement Learning



- S_t = Concept C
- $R_t = \begin{cases} 1 & \text{if } acc(C) = 1 \\ 0 & \text{else} \end{cases}$
- A_t = Transition from concept C to some concept D

Improving Quality Functions

Reinforcement Learning – Q Function

- Maximize

$$G_t = \sum_{i=0}^{n} \gamma^i R_{t+i}$$

Improving Quality Functions

Reinforcement Learning – Q Function

- Maximize

$$G_t = \sum_{i=0}^{n} \gamma^i R_{t+i}$$

- Optimize state-action value function $Q_\pi : S \times A \rightarrow \mathbb{R}$ with

$$Q_\pi(\mathbf{s}, \mathbf{a}) = \mathbb{E}_\pi \left[G_t \mid S_t = \mathbf{s}, A_t = \mathbf{a} \right]$$

Improving Quality Functions

Reinforcement Learning – Q Function

- Maximize

$$G_t = \sum_{i=0}^{n} \gamma^i R_{t+i}$$

- Optimize state-action value function $Q_\pi : S \times A \to \mathbb{R}$ with

$$Q_\pi(\mathbf{s}, \mathbf{a}) = \mathbb{E}_\pi \left[G_t \mid S_t = \mathbf{s}, A_t = \mathbf{a} \right]$$

- Observation: Infinite number of states as search space is infinite

Improving Quality Functions

Reinforcement Learning – Q Function

- Maximize

$$G_t = \sum_{i=0}^{n} \gamma^i R_{t+i}$$

- Optimize state-action value function $Q_\pi : S \times A \rightarrow \mathbb{R}$ with

$$Q_\pi(\mathbf{s}, \mathbf{a}) = \mathbb{E}_\pi \left[G_t \mid S_t = \mathbf{s}, A_t = \mathbf{a}\right]$$

- Observation: Infinite number of states as search space is infinite
- Apply deep Q learning with target network [Mnih et al., 2015]

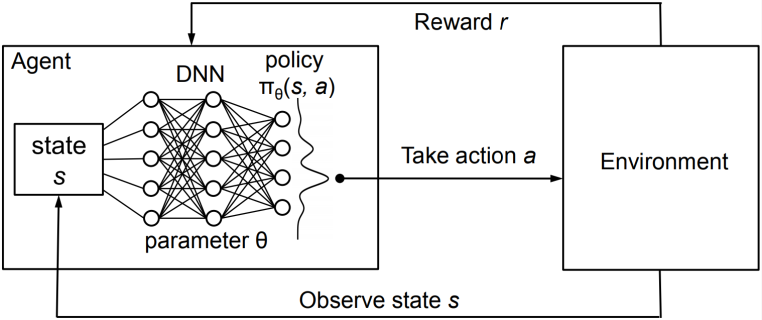
$$\mathcal{L}(\Theta_i) = \mathbb{E}_{(s,a,R,s')\sim U(\mathcal{D})}\left[\left(R + \gamma \max_{\mathbf{a}' \in A(\mathbf{s}')} Q(s', a'; \Theta_i^-) - Q(s, a; \Theta_i)\right)^2\right]$$

Improving Quality Functions

Reinforcement Learning – DRILL

- Convolutional deep Q-Network with $\Theta = [\omega, \mathbf{W}, \mathbf{H}]$

$$\varphi([s, s', \mathbf{e}_+, \mathbf{e}_-]; \Theta) = ReLU\Big(vec(ReLU\big[\Psi([s, s', \mathbf{e}_+, \mathbf{e}_-]) * \omega\big]) \cdot \mathbf{W}\Big) \cdot \mathbf{H}$$



Source: [Mao et al., 2016]

Improving Quality Functions

TransE

- Assumptions
  - Resources and properties are vectors
  - If $(s, p, o) \in E$, then $\vec{s} + \vec{p} = \vec{o}$

Improving Quality Functions

TransE

- Assumptions
  - Resources and properties are vectors
  - If $(s, p, o) \in E$, then $\vec{s} + \vec{p} = \vec{o}$
- Translates to loss

$$L_{pos} = \sum_{(s,p,o) \in E} d(\vec{s} + \vec{p}, \vec{o})$$

Improving Quality Functions

TransE

- Assumptions
  - Resources and properties are vectors
  - If $(s, p, o) \in E$, then $\vec{s} + \vec{p} = \vec{o}$
- Translates to loss

$$L_{pos} = \sum_{(s,p,o) \in E} d(\vec{s} + \vec{p}, \vec{o})$$

- Problem: Loss function converges to trivial solution

Improving Quality Functions

TransE

- Assumptions
  - Resources and properties are vectors
  - If $(s, p, o) \in E$, then $\vec{s} + \vec{p} = \vec{o}$
- Translates to loss

$$L_{pos} = \sum_{(s,p,o) \in E} d(\vec{s} + \vec{p}, \vec{o})$$

- Problem: Loss function converges to trivial solution
- Solution: Add negative information and margin $\gamma \in \mathbb{R}^+$
- Loss is now

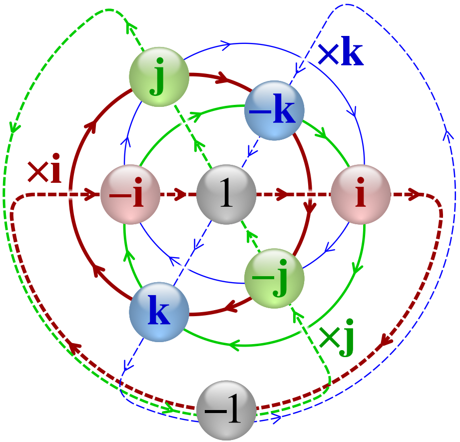
$$L = \sum_{(s,p,o) \in E} \sum_{(s',p,o') \in S'(s,p,o)} [\gamma + d(\vec{s} + \vec{p}, \vec{o}) - d(\vec{s'} + \vec{p}, \vec{o'})]_+$$

where

- $S'(s, p, o) = sample(\{(s', p, o) | s' \in V\} \cup \{(s, p, o') | o' \in V\}, 1)$
- $S'(s, p, o) \cap E = \emptyset$
- $[x]_+ = \max\{0, x\}$

Improving Quality Functions

Quaternions: $\mathbb{H}$



2

[2] https://en.wikipedia.org/wiki/Quaternion#/media/File:Cayley_Q8_quaternion_multiplication_graph.svg

Improving Quality Functions

Quaternions: $\mathbb{H}$

- Can define embeddings in this space: QMult [Demir et al., 2021]
  - $\vec{s}, \vec{p}, \vec{o} \in \mathbb{H}^k$
  - Scoring function $\varphi(s, p, o) = (\vec{s} \otimes \vec{p}) \cdot \vec{o}$, where

Improving Quality Functions

Quaternions: $\mathbb{H}$

- Can define embeddings in this space: QMult [Demir et al., 2021]
  - $\vec{s}, \vec{p}, \vec{o} \in \mathbb{H}^k$
  - Scoring function $\varphi(s, p, o) = (\vec{s} \otimes \vec{p}) \cdot \vec{o}$, where
    - $\otimes$ is the Hamiltonian product ($\mathbb{H} \times \mathbb{H} \rightarrow \mathbb{H}$)
    - $\cdot$ is the quaternion inner product ($\mathbb{H} \times \mathbb{H} \rightarrow \mathbb{R}$)

Improving Quality Functions

Quaternions: $\mathbb{H}$

- Can define embeddings in this space: QMult [Demir et al., 2021]
  - $\vec{s}, \vec{p}, \vec{o} \in \mathbb{H}^k$
  - Scoring function $\varphi(s, p, o) = (\vec{s} \otimes \vec{p}) \cdot \vec{o}$, where
    - $\otimes$ is the Hamiltonian product ($\mathbb{H} \times \mathbb{H} \rightarrow \mathbb{H}$)
    - $\cdot$ is the quaternion inner product ($\mathbb{H} \times \mathbb{H} \rightarrow \mathbb{R}$)
  - Loss function over training data Γ with $Y_{spo} \in \{-1, +1\}$ is given by
    $$\sum_{(s,p,o)\in\Gamma} \log(1 + exp(-Y_{spo}\varphi(s, p, o)))$$

Improving Quality Functions

Quaternions: $\mathbb{H}$

- Can define embeddings in this space: QMult [Demir et al., 2021]
  - $\vec{s}, \vec{p}, \vec{o} \in \mathbb{H}^k$
  - Scoring function $\varphi(s, p, o) = (\vec{s} \otimes \vec{p}) \cdot \vec{o}$, where
    - $\otimes$ is the Hamiltonian product ($\mathbb{H} \times \mathbb{H} \to \mathbb{H}$)
    - $\cdot$ is the quaternion inner product ($\mathbb{H} \times \mathbb{H} \to \mathbb{R}$)
  - Loss function over training data Γ with $Y_{spo} \in \{-1, +1\}$ is given by
    $$\sum_{(s,p,o)\in\Gamma} \log(1 + exp(-Y_{spo}\varphi(s, p, o)))$$
- Similar construction for octonions

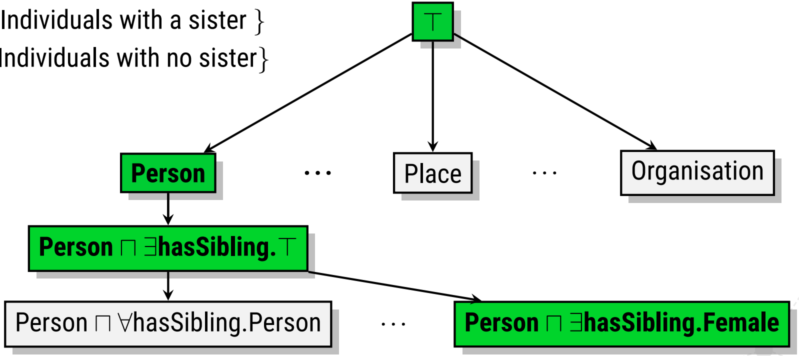
Improving Quality Functions

Unsupervised Learning – Training Data

- Follow refinement path at random
- Select concept C
- Set $E^+ \subseteq R(C)$ and $E^- \cap R(C) = \emptyset$

$E^+ = \{$Individuals with a sister $\}$

$E^- = \{$Individuals with no sister$\}$



Improving Quality Functions

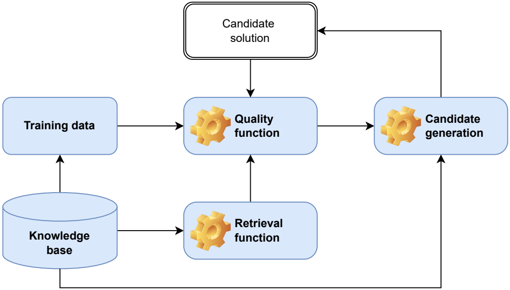
Evaluation

- Used Family und BioPax datasets
- Evaluation on 114 learning problems

Approaches	F1	Acc	Runtime	# Exp.
CELOE	$.995 \pm 0.03$	$.993 \pm 0.04$	7.5 ± 1.1	33.5 ± 129.3
OCEL	*	1.00 ± 0.00	11.0 ± 1.4	2271.6 ± 1269.2
ELTL	$.990 \pm 0.06$	$.984 \pm 0.09$	8.1 ± 1.6	*
DRILL	1.00 ± 0.00	1.00 ± 0.00	1.1 ± 0.5	9.88 ± 38.5

Learning problem

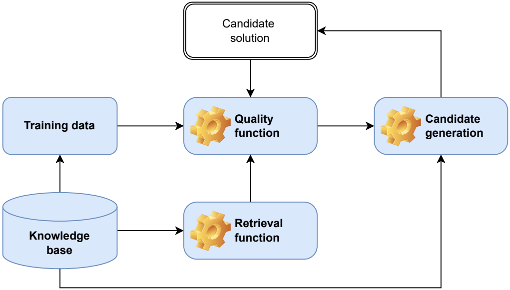
Challenges



✓ Retrieval is expensive

Learning problem

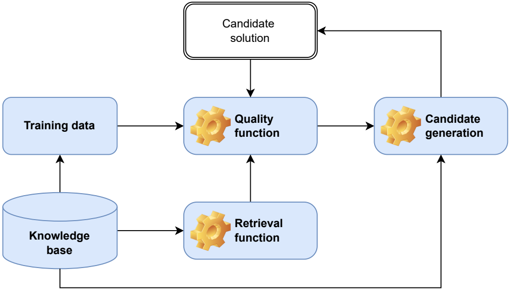
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic

Learning problem

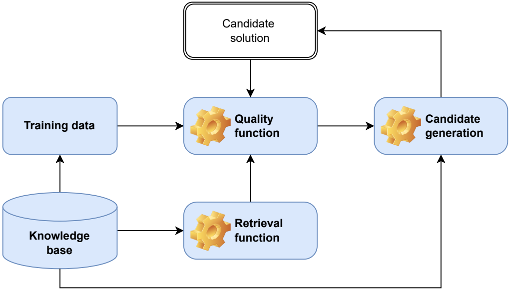
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic ⇒ Exploit embeddings
- ► Candidate generation is expensive

Learning problem

Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic ⇒ Exploit embeddings
- ► Candidate generation is expensive ⇒ Exploit priming
- ► Search space is large ⇒ Prune by length

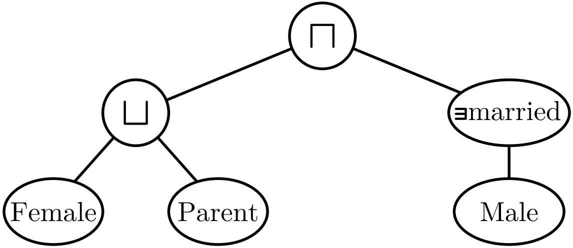
Section 5

Learning with Priming

Learning with Priming

EvoLearner – Idea

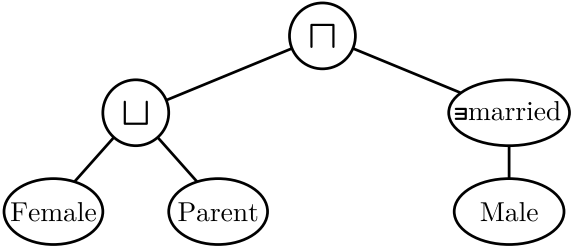
- Represent concepts as trees, e.g.,
  $(\texttt{Female} \sqcup \texttt{Parent}) \sqcap \exists \texttt{married.Male}$



Learning with Priming

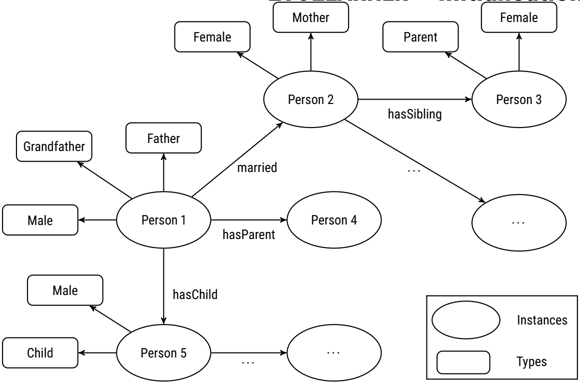
EvoLearner – Idea

- Represent concepts as trees, e.g.,
  (Female ⊔ Parent) ⊓ ∃married.Male
- Learn in evolutionary fashion using genetic programming
- Exploit priming effect (remember the green apple)
- Intuition: An individual is an overlap several concepts [Heindorf et al., 2022]



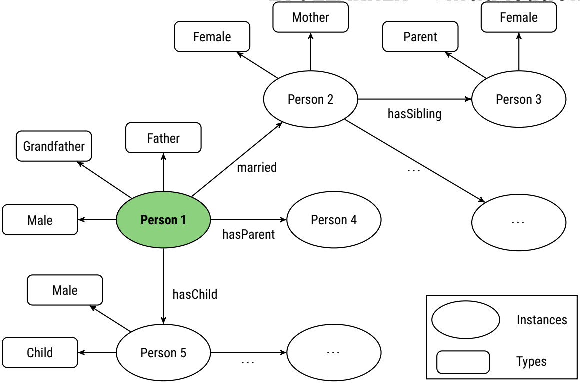
Learning with Priming

EvoLearner – Initialisation



Learning with Priming

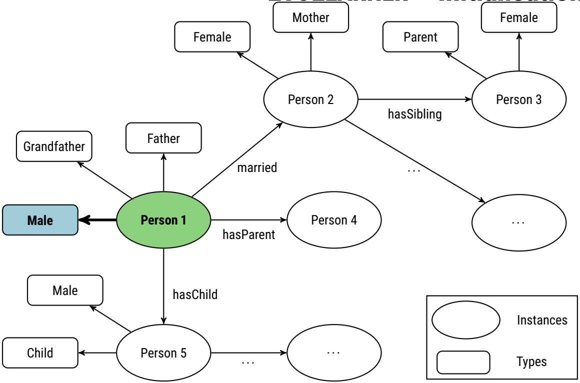
EvoLearner – Initialisation



1. Select a positive example e^+ and one of its types:

Learning with Priming

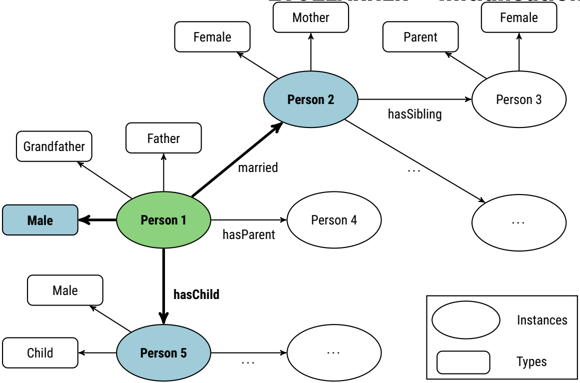
EvoLearner – Initialisation



1. Select a positive example e^+ and one of its types: `Male`

Learning with Priming

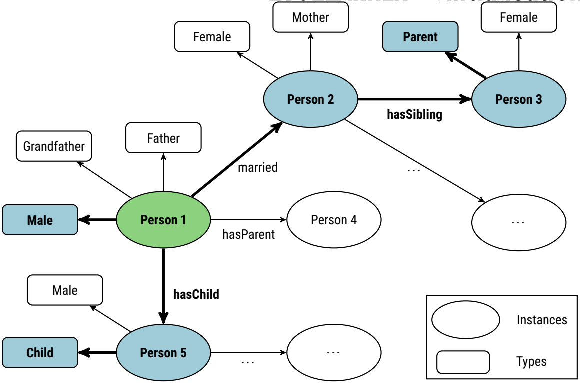
EvoLearner – Initialisation



1. Select a positive example e^+ and one of its types: `Male`
2. Randomly select up to *maxT* outgoing triples of e^+:
   $\texttt{Male} \sqcap (\exists\texttt{married} \ldots \sqcap \exists\texttt{hasChild} \ldots)$

Learning with Priming

EvoLearner – Initialisation



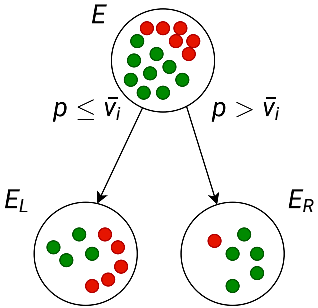
1. Select a positive example e^+ and one of its types: `Male`
2. Randomly select up to *maxT* outgoing triples of e^+:
   $\texttt{Male} \sqcap (\exists\texttt{married} \ldots \sqcap \exists\texttt{hasChild} \ldots)$
3. Complete incomplete subconcepts:
   $\texttt{Male} \sqcap ((\exists\texttt{married}.\ \exists\texttt{hasSibling}.\texttt{Parent}) \sqcap (\exists\texttt{hasChild}.\texttt{Child}))$

Learning with Priming

EvoLearner – Data Properties

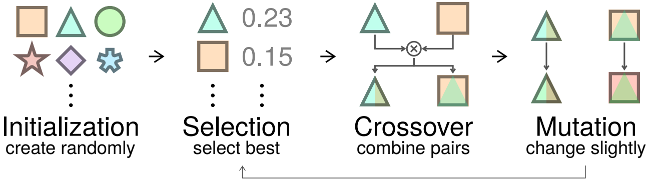
- Given a data property d from the knowledge base $\mathcal{K}$ and a set E of positive and negative examples
- We precompute up to k splits of the form $d \leq \bar{v}_i$ per data property
- Splits are computed to maximize information gain:

$$IG(E, \bar{v}_i) = H(E) - H(E|\bar{v}_i) = H(E) - \left(\frac{|E_L|}{|E|} H(E_L) + \frac{|E_R|}{|E|} H(E_R) \right)$$



Learning with Priming

EvoLearner – Training



Learning with Priming

EvoLearner – Evaluation

Learn. Problem	EvoLearner (ours)	DL-Learner (CELOE)	DL-Learner (OCEL)	Aleph	SPaCEL
Carcinogenesis	0.70 ± 0.12	**0.71** $\pm$ **0.01**	*no results*	0.46 ± 0.12	0.60 ± 0.08
Family	1.00 ± 0.01	0.98 ± 0.05	**1.00** $\pm$ **0.00**	–	0.97 ± 0.11
Hepatitis	**0.79** $\pm$ **0.08**	0.61 ± 0.03	*no results*	0.38 ± 0.12	*no results*
Lymphography	0.84 ± 0.09	0.78 ± 0.10	**0.85** $\pm$ **0.10**	0.84 ± 0.09	0.75 ± 0.13
Mammographic	**0.81** $\pm$ **0.06**	0.64 ± 0.01	0.78 ± 0.08	0.48 ± 0.08	0.64 ± 0.06
Mutagenesis	**1.00** $\pm$ **0.00**	0.93 ± 0.14	*timeout*	0.43 ± 0.47	**1.00** $\pm$**0.00**
NCTRER	**1.00** $\pm$ **0.00**	0.74 ± 0.01	0.94 ± 0.06	0.71 ± 0.18	**1.00** $\pm$ **0.00**
Premier League	**1.00** $\pm$ **0.00**	0.99 ± 0.04	0.81 ± 0.13	0.94 ± 0.11	0.98 ± 0.04
Pyrimidine	**0.91** $\pm$ **0.14**	0.84 ± 0.15	0.84 ± 0.22	0.90 ± 0.32	0.86 ± 0.29

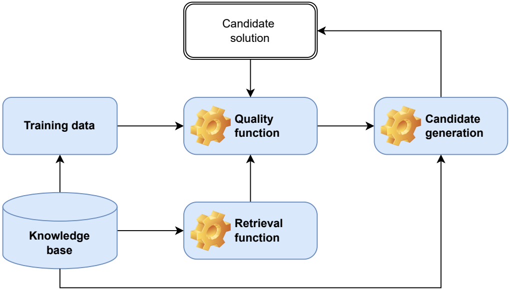
Learning with Priming

EvoLearner – Ablation Study

Learning Problem	EvoLearner (ours)	Without Rand. Walk Init.	Without Data Properties	Without Both
Carcinogenesis	0.70 ± 0.12	0.60 ± 0.21	0.63 ± 0.13	0.62 ± 0.13
Family	1.00 ± 0.01	0.87 ± 0.13	–	0.86 ± 0.14
Hepatitis	0.79 ± 0.08	0.67 ± 0.15	0.46 ± 0.14	0.47 ± 0.13
Lymphography	0.84 ± 0.09	0.83 ± 0.11	–	0.83 ± 0.09
Mammographic	0.81 ± 0.06	0.78 ± 0.08	0.77 ± 0.07	0.75 ± 0.06
Mutagenesis	1.00 ± 0.00	1.00 ± 0.00	0.44 ± 0.48	0.50 ± 0.51
NCTRER	1.00 ± 0.00	1.00 ± 0.00	0.74 ± 0.05	0.75 ± 0.05
Premier League	1.00 ± 0.00	0.98 ± 0.04	0.50 ± 0.23	0.50 ± 0.22
Pyrimidine	0.91 ± 0.14	0.83 ± 0.22	0.67 ± 0.00	0.67 ± 0.00

Learning problem

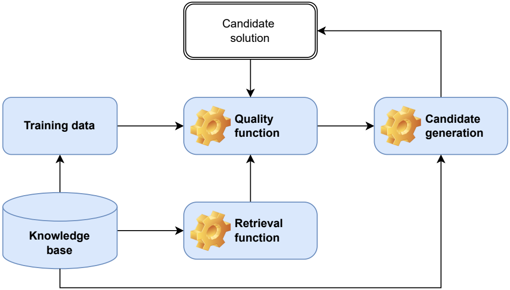
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic

Learning problem

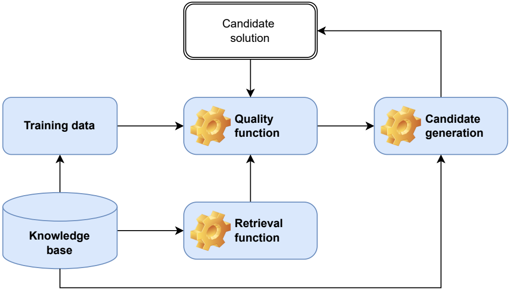
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic ⇒ Exploit embeddings
- ✓ Candidate generation is expensive

Learning problem

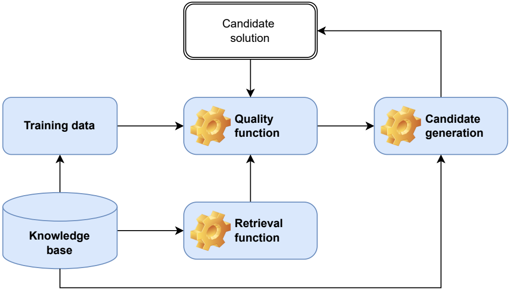
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic ⇒ Exploit embeddings
- ✓ Candidate generation is expensive ⇒ Exploit priming

Learning problem

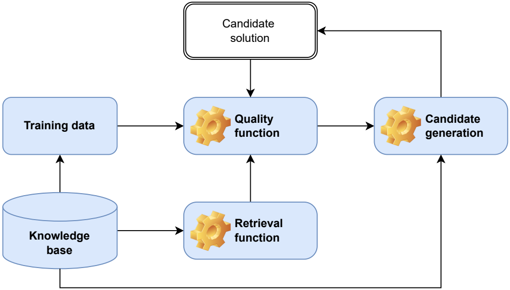
Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic ⇒ Exploit embeddings
- ✓ Candidate generation is expensive ⇒ Exploit priming
- ▶ Search space is large

Learning problem

Challenges



- ✓ Retrieval is expensive ⇒ Exploit SPARQL
- ✓ Quality functions are often myopic ⇒ Exploit embeddings
- ✓ Candidate generation is expensive ⇒ Exploit priming
- ► Search space is large ⇒ Prune by length

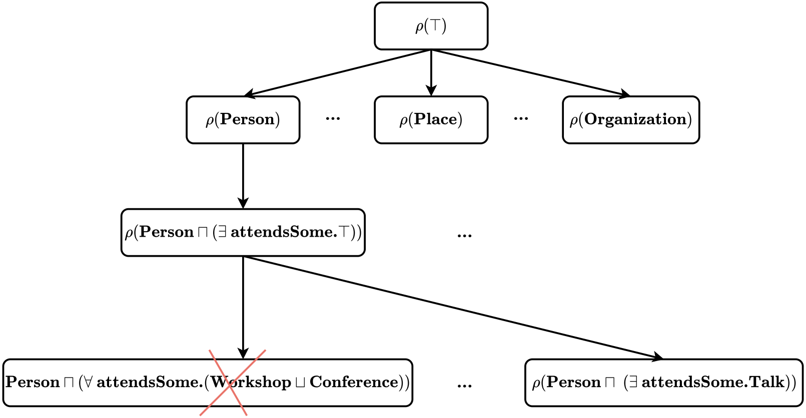
Section 6

CLIP

CLIP

Approach

- Idea: Prune horizontally by
- predicting target concept length and
- discarding longer refinements



Concept Lengths

- $length(A) = length(\top) = length(\bot) = 1$ (if A is an atomic concept)

Concept Lengths

- $length(A) = length(\top) = length(\bot) = 1$ (if A is an atomic concept)
- $length(\neg C) = 1 + length(C)$, for all concepts C

CLIP

Concept Lengths

- $length(A) = length(\top) = length(\bot) = 1$ (if A is an atomic concept)
- $length(\neg C) = 1 + length(C)$, for all concepts C
- $length(\exists\, r.C) = length(\forall\, r.C) = 2 + length(C)$, for all concepts C

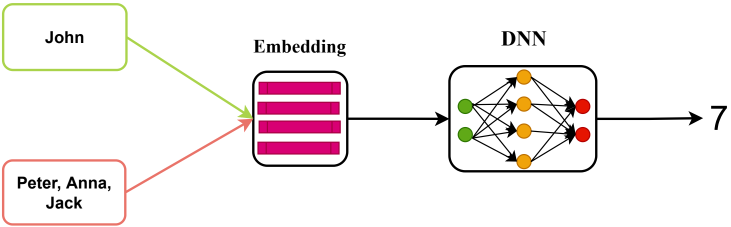
CLIP

Concept Lengths

- $length(A) = length(\top) = length(\bot) = 1$ (if A is an atomic concept)
- $length(\neg C) = 1 + length(C)$, for all concepts C
- $length(\exists\, r.C) = length(\forall\, r.C) = 2 + length(C)$, for all concepts C
- $length(C \sqcup D) = length(C \sqcap D) = 1 + length(C) + length(D)$, for all concepts C and D.

CLIP

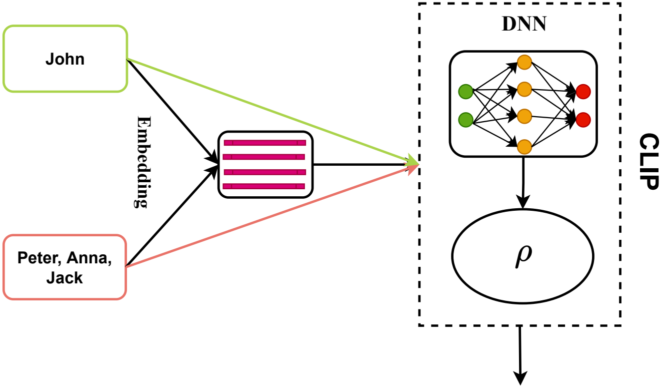
Concept Length Prediction



- Input: positive and negative examples
- Output: length of the target concept

CLIP

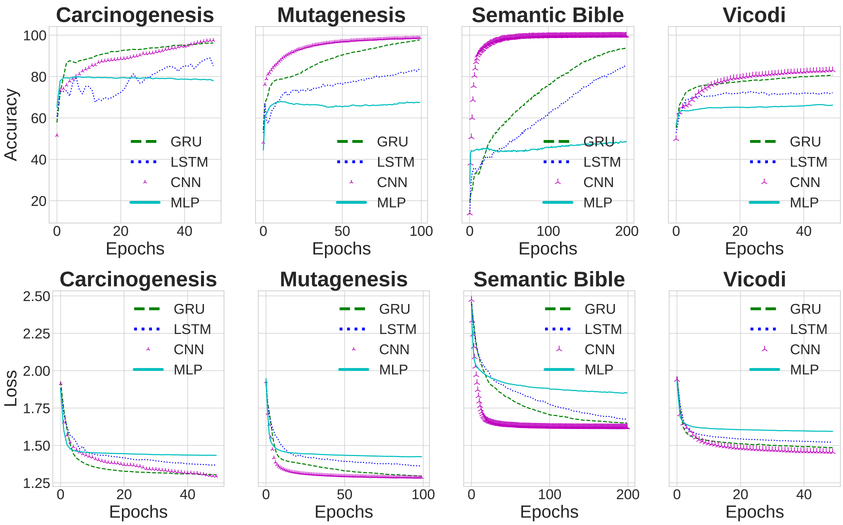
Concept Learning



Male ⊓ ∃ hasParent.(∃ hasChild.Female)

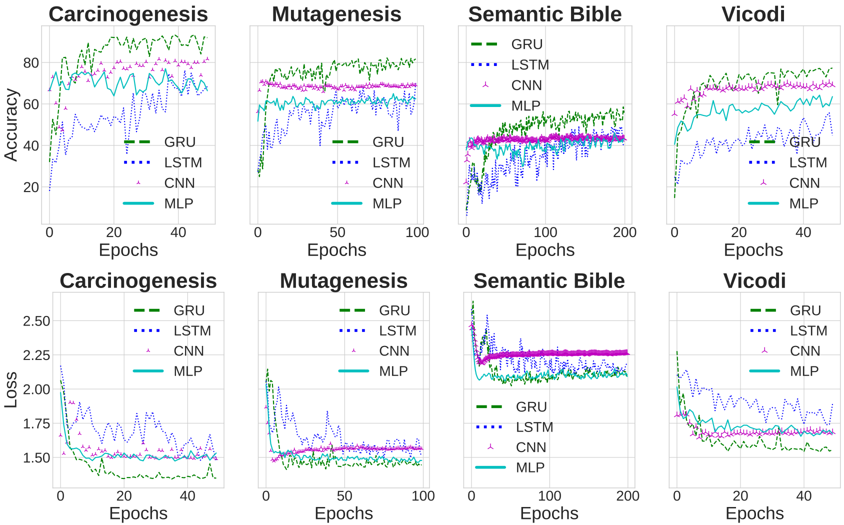
CLIP

Training



CLIP

Validation



CLIP

Network Architecture

	Carcinogenesis					Mutagenesis				
Metric	**LSTM**	**GRU**	**CNN**	**MLP**	**RM**	**LSTM**	**GRU**	**CNN**	**MLP**	**RM**
Train. Acc.	0.89	0.96	0.97	0.80	0.48	0.83	0.97	0.98	0.68	0.33
Val. Acc.	0.76	0.93	0.82	0.77	0.48	0.70	0.82	0.71	0.65	0.35
Test Acc.	0.92	**0.95**	0.84	0.80	0.49	0.78	**0.85**	0.70	0.68	0.33
Test F1	0.88	**0.92**	0.71	0.59	0.33	0.76	**0.85**	0.70	0.67	0.32

	Semantic Bible					Vicodi				
Metric	**LSTM**	**GRU**	**CNN**	**MLP**	**RM**	**LSTM**	**GRU**	**CNN**	**MLP**	**RM**
Train. Acc.	0.85	0.93	0.99	0.68	0.33	0.73	0.81	0.83	0.66	0.28
Val. Acc.	0.49	0.58	0.44	0.46	0.26	0.55	0.77	0.70	0.64	0.30
Test Acc.	0.52	**0.53**	0.37	0.40	0.25	0.66	**0.80**	0.69	0.66	0.29
Test F1	0.27	**0.38**	0.20	0.22	0.16	0.45	**0.50**	0.45	0.38	0.20

CLIP

Comparison with SOTA

Carcinogenesis				
Metric	**CELOE**	**OCEL**	**ELTL**	**CLIP**
Acc. ↑	0.78 ± 0.27	0.89 ± 0.31	0.58 ± 0.46	**0.99** ± 0.00
F1 ↑	0.62 ± 0.46	–	0.51 ± 0.47	**0.96**∗ ± 0.10
Runtime (min) ↓	0.93 ± 0.94	3.01 ± 0.72	0.75 ± 0.07	**0.10**∗ ± 0.09
Length ↓	**1.69** ± 0.89	7.81 ± 6.88	1.04 ± 0.39	2.00 ± 1.28

Mutagenesis				
Metric	**CELOE**	**OCEL**	**ELTL**	**CLIP**
Acc. ↑	0.99 ± 0.00	0.71 ± 0.45	0.37 ± 0.43	**0.99** ± 0.00
F1 ↑	0.81 ± 0.35	–	0.29 ± 0.40	**0.93**∗ ± 0.18
Runtime (min) ↓	0.70 ± 0.77	2.39 ± 0.18	0.29 ± 0.16	**0.07**∗ ± 0.05
Length ↓	2.79 ± 1.17	12.63 ± 7.03	1.10 ± 0.81	**2.20** ± 1.16

Semantic Bible				
Metric	**CELOE**	**OCEL**	**ELTL**	**CLIP**
Acc. ↑	0.99 ± 0.02	0.66 ± 0.47	0.59 ± 0.37	**0.99** ± 0.00
F1 ↑	0.97 ± 0.10	–	0.57 ± 0.38	**0.98** ± 0.05
Runtime (min) ↓	0.47 ± 0.80	22.15 ± 96.55	0.09 ± 0.07	**0.06**∗ ± 0.05
Length ↓	3.85 ± 2.44	9.54 ± 5.73	1.38 ± 1.76	**2.52**∗ ± 1.26

Vicodi				
Metric	**CELOE**	**OCEL**	**ELTL**	**CLIP**
Acc. ↑	0.29 ± 0.44	0.25 ± 0.43	0.28 ± 0.44	**0.99**∗ ± 0.00
F1 ↑	0.25 ± 0.44	–	0.25 ± 0.44	**0.97**∗ ± 0.09
Runtime (min) ↓	1.30 ± 0.71	4.78 ± 1.12	1.81 ± 0.46	**0.16**∗ ± 0.12
Length ↓	10.79 ± 6.30	11.54 ± 6.00	11.14 ± 6.11	**1.68**∗ ± 0.98

Section 7

Summary

Summary

Open Questions

- Tensors: Variable ordering? Compressed data structure?
- RL: Reduce training costs? Hyperparameters? Embeddings?
- Evolutionary learning: Myopia? Runtime? Continuous data?

Summary

Open Questions

Holy Grail

- Can the selection of representations be automated?
- LEMUR and ENEXA

- Tensors: Variable ordering? Compressed data structure?
- RL: Reduce training costs? Hyperparameters? Embeddings?
- Evolutionary learning: Myopia? Runtime? Continuous data?

Summary

Thank You!

Joint works with Alexander Bigerl, Caglar Demir, Hamada Zahera, N'Dah Jean Kouagou, Nikoloas Karalis, Stefan Heindorf, Mohamed Sherif, Muhammed Saleem, and many more

Thank You!

Questions?

- https://dice-research.org
- https://twitter.com/DiceResearch
- https://twitter.com/NgongaAxel

References I

[Barr, 1989] Barr, A. H. (1989).
The einstein summation notation: Introduction and extensions.
SIGGRAPH 89 Course notes# 30 on Topics in Physically-Based Modeling, pages J1–J12.

[Bigerl et al., 2020] Bigerl, A., Conrads, F., Behning, C., Sherif, M. A., Saleem, M., and Ngonga Ngomo, A.-C. (2020).
Tentris–a tensor-based triple store.
In *International Semantic Web Conference*, pages 56–73. Springer.

[Bigerl et al., 2022] Bigerl, A., Conrads, L., Behning, C., Saleem, M., and Ngonga Ngomo, A.-C. (2022).
Hashing the hypertrie: Space-and time-efficient indexing for sparql in tensors.
In *International Semantic Web Conference*, pages 57–73. Springer.

[Bin et al., 2016] Bin, S., Bühmann, L., Lehmann, J., and Ngonga Ngomo, A.-C. (2016).
Towards sparql-based induction for large-scale rdf data sets.
In *ECAI 2016*, pages 1551–1552. IOS Press.

[Demir et al., 2021] Demir, C., Moussallem, D., Heindorf, S., and Ngomo, A.-C. N. (2021).
Convolutional hypercomplex embeddings for link prediction.
In *Asian Conference on Machine Learning*, pages 656–671. PMLR.

[Demir and Ngonga Ngomo, 2021] Demir, C. and Ngonga Ngomo, A.-C. (2021).
Drill–deep reinforcement learning for refinement operators in *alc*.
arXiv preprint arXiv:2106.15373.

References III

[Heindorf et al., 2022] Heindorf, S., Blübaum, L., Düsterhus, N., Werner, T., Golani, V. N., Demir, C., and Ngonga Ngomo, A.-C. (2022).
Evolearner: Learning description logics with evolutionary algorithms.
In *Proceedings of the ACM Web Conference 2022*, pages 818–828.

[Kahneman, 2011] Kahneman, D. (2011).
Thinking, fast and slow.
Macmillan.

[Lehmann and Hitzler, 2010] Lehmann, J. and Hitzler, P. (2010).
Concept learning in description logics using refinement operators.
Machine Learning, 78(1):203–250.

References IV

[Mao et al., 2016] Mao, H., Alizadeh, M., Menache, I., and Kandula, S. (2016).
Resource management with deep reinforcement learning.
In *Proceedings of the 15th ACM workshop on hot topics in networks*, pages 50–56.

[Mnih et al., 2015] Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., Graves, A., Riedmiller, M., Fidjeland, A. K., Ostrovski, G., et al. (2015).
Human-level control through deep reinforcement learning.
nature, 518(7540):529–533.

[Schmidt-Schauß and Smolka, 1991] Schmidt-Schauß, M. and Smolka, G. (1991).
Attributive concept descriptions with complements.
Artificial intelligence, 48(1):1–26.