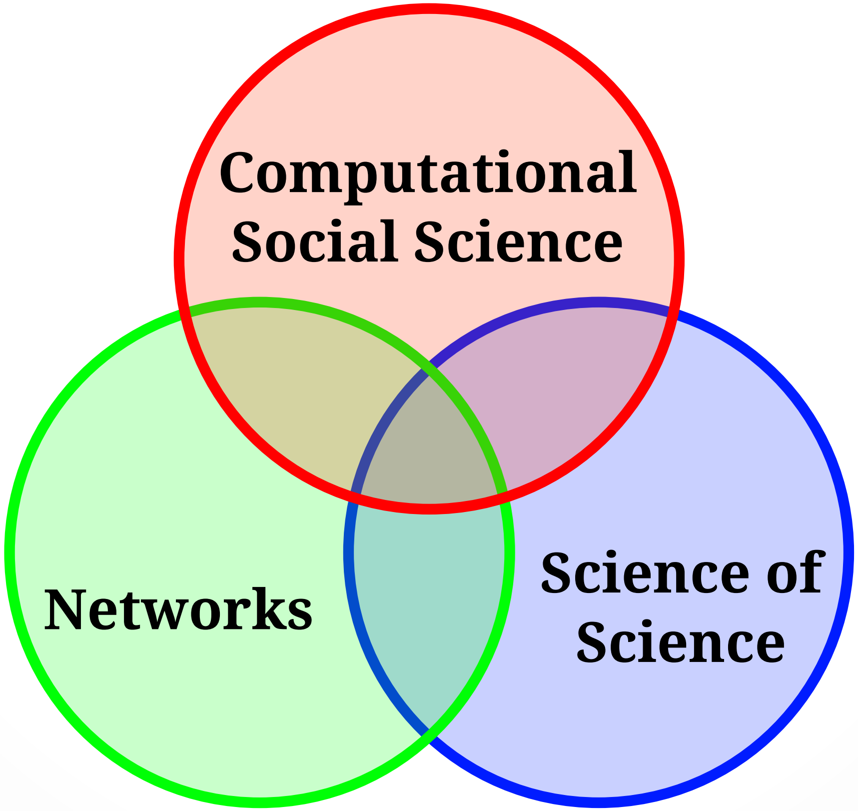
Community structure in complex networks

Santo Fortunato

INDIANA UNIVERSITY

Me in a nutshell…



Computational social science

REVIEWS OF MODERN PHYSICS, VOLUME 81, APRIL–JUNE 2009

Statistical physics of social dynamics

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SMC, INFM-CNR and Dipartimento di Fisica, "Sapienza" Università di Roma, Piazzale A. Moro 2, 00185 Roma, Italy

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(Published 11 May 2009)

Statistical physics has proven to be a fruitful framework to describe phenomena outside the realm of traditional physics. Recent years have witnessed an attempt by physicists to study collective phenomena emerging from the interactions of individuals as elementary units in social structures. A wide list of topics are reviewed ranging from opinion and cultural and language dynamics to crowd behavior, hierarchy formation, human dynamics, and social spreading. The connections between these problems and other, more traditional, topics of statistical physics are highlighted. Comparison of model results with empirical data from social systems are also emphasized.

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CONTENTS

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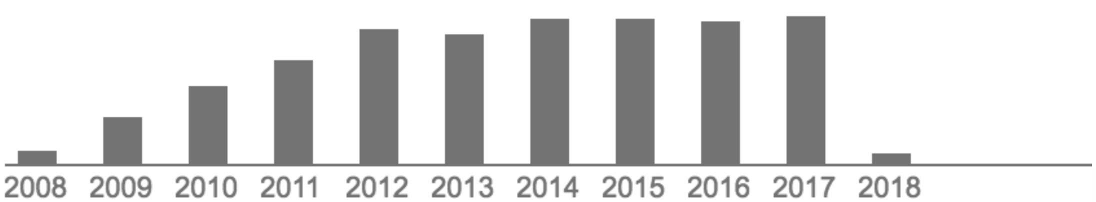

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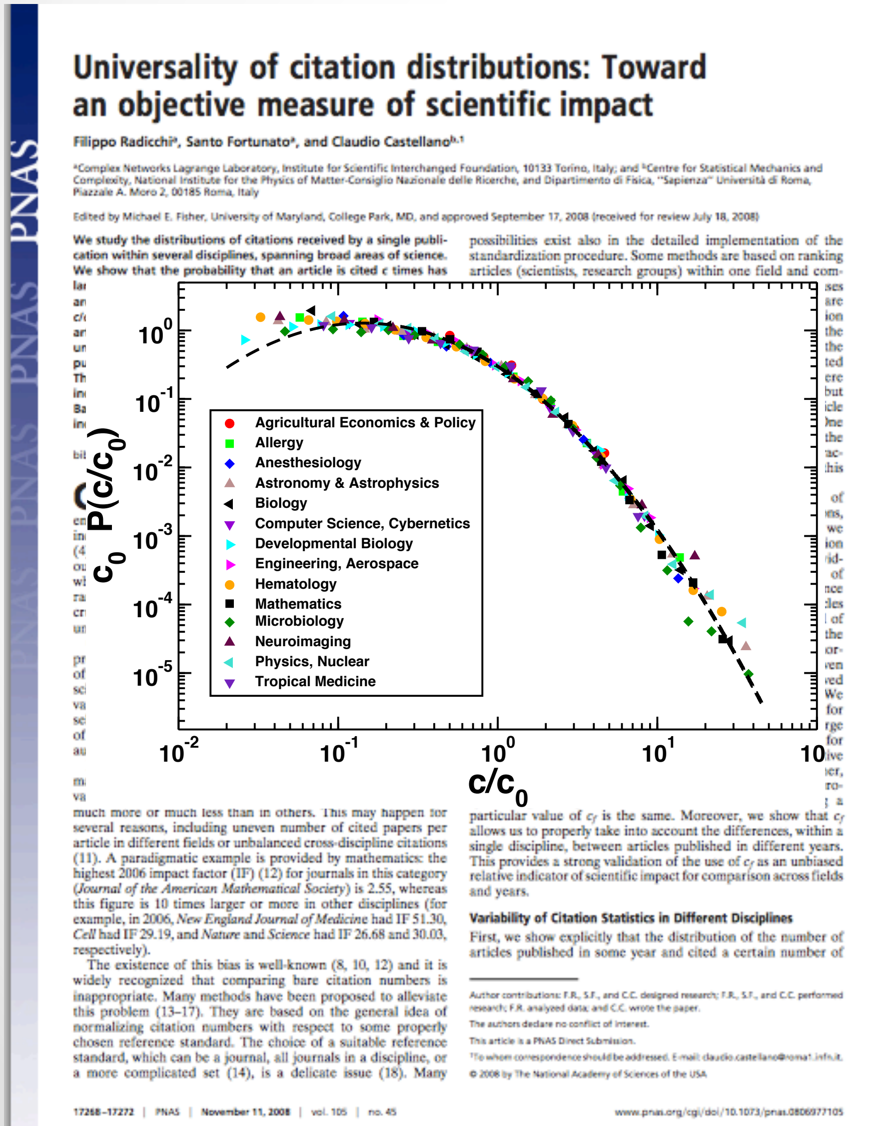
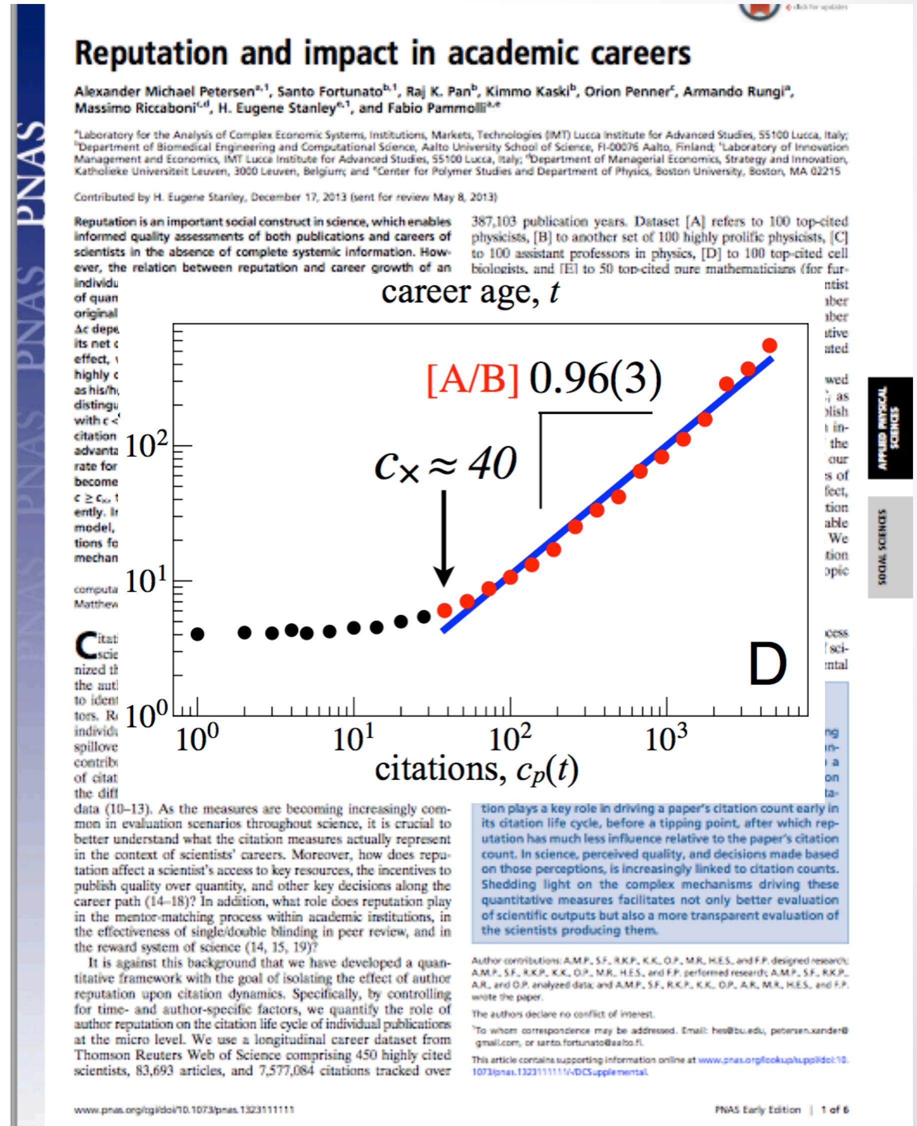
Science of science





Science of science

Correspondence

Growing time lag threatens Nobels

The time lag between reporting a scientific discovery worthy of a Nobel prize and the awarding of the medal has increased, with waits of more than 20 years becoming common. If this trend continues, some candidates might not live long enough to attend their Nobel ceremonies.

Before 1940, Nobels were awarded more than 20 years after the original discovery for only about 11% of physics, 15% of chemistry and 24% of physiology or medicine prizes, respectively. Since 1985, however, such lengthy delays have featured in 60%, 52% and 45% of these awards, respectively.

The increasing average interval between reporting discoveries and their formal recognition can be fitted to an exponential curve (see 'The long road to Sweden'), with data points scattered about the mean value.

As this average interval becomes longer, so the average age at which laureates are awarded the prize goes up. By the end of this century, the prizewinners' predicted average age for receiving the award is likely to exceed his or her projected life expectancy (data not shown). Given that the Nobel prize cannot be awarded posthumously, this lag threatens to undermine science's most venerable institution.

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**On behalf of 6 co-authors; see go.nature.com/cmmxa5 for full list.*

THE LONG ROAD TO SWEDEN
Scientists who publish prizewinning discoveries are, on average, waiting longer for a Nobel than ever before.
Physics
Chemistry
Physiology or medicine
Years since discovery
0
10
20
30
40
1920
1960
2000
Year of Nobel prize

Livestock: tackle demand and yields

Among many otherwise laudable suggestions, Mark Eisler and colleagues propose limiting feedstuffs for livestock to fibrous fodder, such as grass and silage (see *Nature* **507**, 32–34; 2014). However, we believe that any attempt to meet the rapid growth in world demand for meat and dairy products by focusing on ruminant grazing systems would be damaging for biodiversity and for the global climate.

Although ruminants convert grass and silage into animal protein, they do so inefficiently; they therefore require much more land to produce a given amount of meat or milk than ruminants fed on diets that include grain. Growing enough fodder to satisfy demand would require the large-scale expansion of grazing lands (see go.nature.com/7mf63y) — a leading cause of biodiversity loss, tropical deforestation and carbon dioxide emissions.

The environmental impacts of meat and dairy production should instead be addressed by stringent efforts to decrease consumption, halt the expansion of grazing, and increase yields on land that is already used for livestock. Promoting extensive grazing without tackling demand would do more harm than good.

Erasmus K. H. J. zu Ermgassen, David R. Williams, Andrew Balmford *University of Cambridge, UK.*
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Livestock: limit red meat consumption

Mark Eisler and co-authors advocate eating only 300 grams of red meat a week (roughly the volume of three decks of playing cards) as a step towards producing sustainable livestock (*Nature* **507**, 32–34; 2014). That amount corresponds to 3.5–7% of a 2,000-calorie-a-day diet, depending on the cut and type of meat. Such a move would also make for a more equitable global distribution of animal-product consumption; these products comprise around 48% of the average diet in the United States, for example (S. Bonhommeau *et al. Proc. Natl Acad. Sci. USA* **110,** 20617–20620; 2013).

Imposing a global dietary limit of 5% red meat as part of a 10% maximum for all animal-based products would enable more people to be fed using less land. For example, eliminating livestock and using existing agricultural lands to grow crops for direct human consumption instead of for livestock fodder could feed an extra 4 billion people (E. S. Cassidy *et al. Environ. Res. Lett.* **8,** 034015; 2013), thereby reducing or eliminating the greenhouse-gas emissions and biodiversity loss associated with conversion of natural habitats. This would also reduce many other environmental impacts of agriculture that relate to the use of water, fertilizer and fossil fuels.

Brian Machovina, Kenneth J. Feeley *Florida International University, Miami; and The Fairchild Tropical Botanic Garden, Coral Gables, Florida, USA.*
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Zoo visits boost biodiversity literacy

Zoos and aquaria worldwide attract more than 700 million visits every year. They are therefore well placed to make more people aware of the importance of biodiversity — a prime target of the United Nations Strategic Plan for Biodiversity 2011–20.

We surveyed approximately 6,000 visitors to 30 zoos and aquaria in 19 countries (see go.nature.com/vwf8yf). More respondents showed improved understanding of biodiversity after their visit (75.1% compared with 69.8% before) and more could identify an individual action that would bolster biodiversity after their visit (58.8% compared with 50.5% before).

Regrettably, increased awareness does not necessarily change behaviour. The world's zoo and aquarium communities must also help to drive important behavioural and social changes to assist conservation.

Andrew Moss *Chester Zoo, UK.*
Eric Jensen *University of Warwick, Coventry, UK.*
Markus Gusset *World Association of Zoos and Aquariums, Gland, Switzerland.*
markus.gusset@waza.org

A protein that spells trouble

The gene *CYLD* is so named because one of its mutant forms is associated with cylindromatosis, which causes skin tumours.

The CYLD protein is an enzyme; its active site in humans contains a cysteine residue at position 601 (denoted as C in the one-letter amino-acid code). The amino-acid sequence following this cysteine (C) is tyrosine (Y), leucine (L) and aspartate (D). What are the odds of that?

David Boone *Indiana University School of Medicine — South Bend, Indiana, USA.*
daboone@iu.edu

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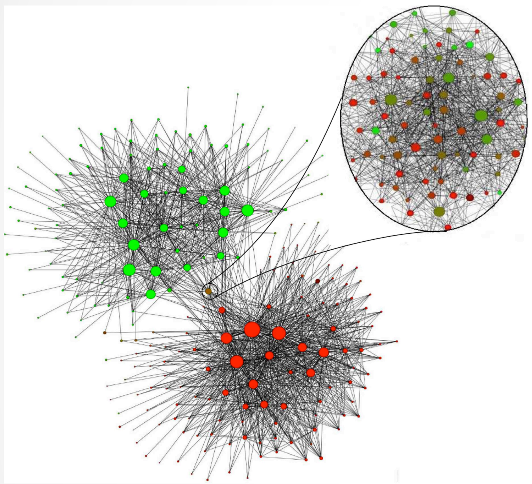
Network science

- **Analysis and modeling**

Social



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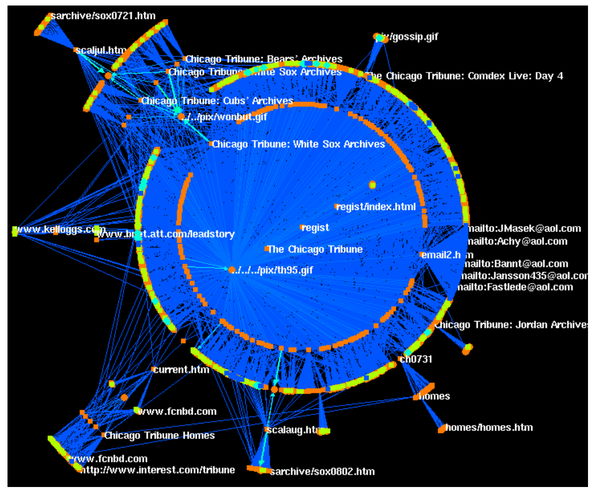

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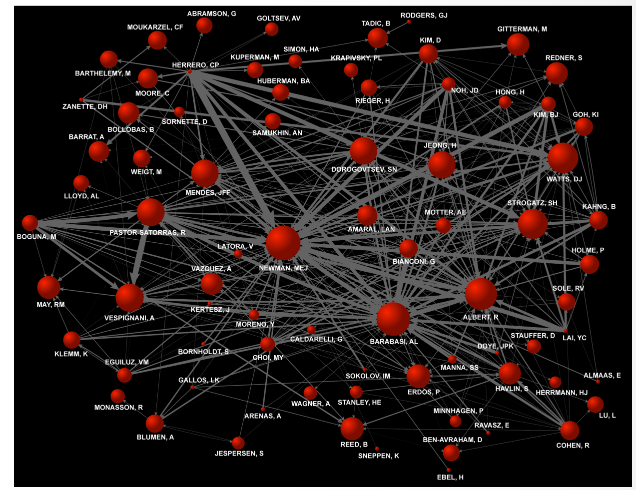
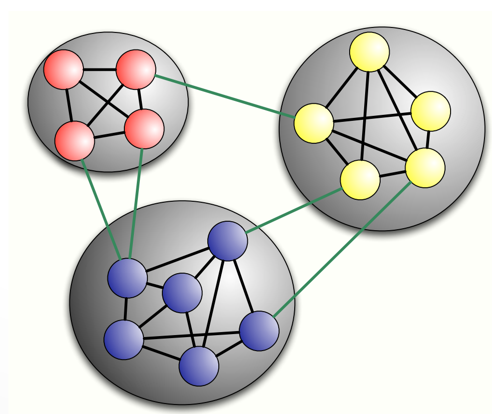


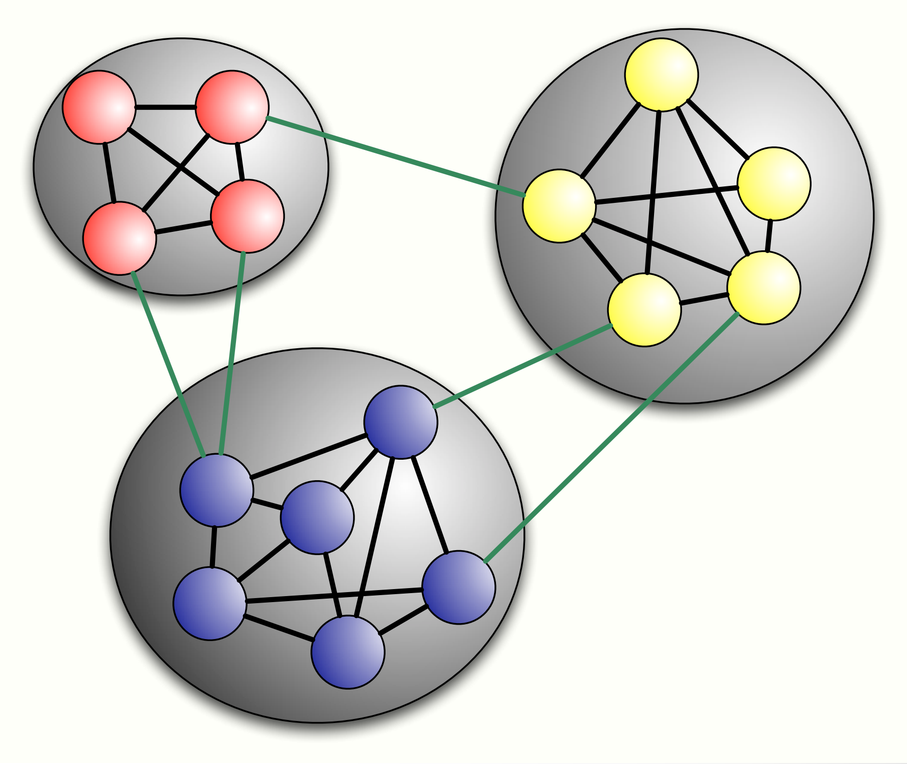
- **Community structure**



Community structure

Communities: sets of tightly connected nodes

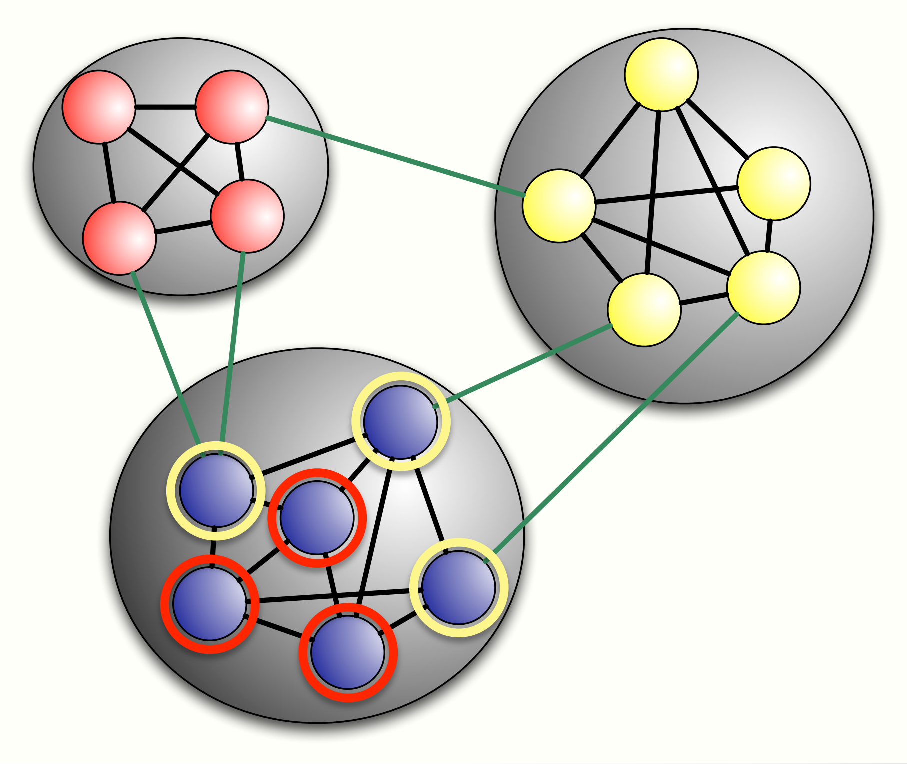
- People with common interests
- Scholars working on the same field
- Proteins with equal/similar functions
- Papers on the same/related topics
- …



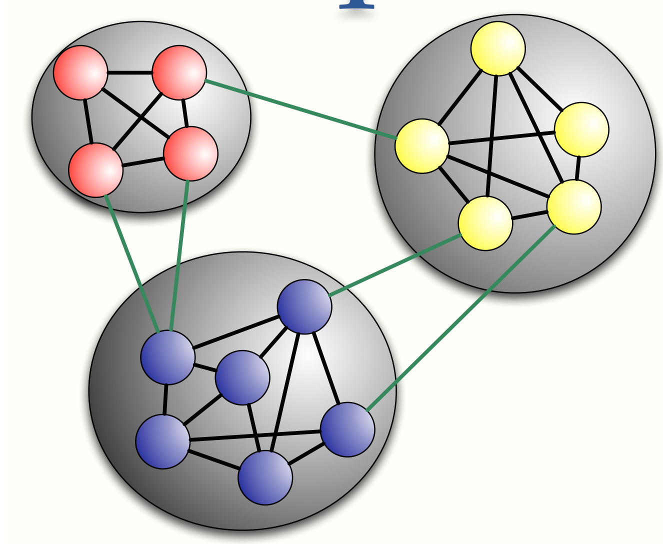
Community detection

What for?

- Organization
- Node classification
- Missing links
- Effect on dynamics
- …



Difficult problem!



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Difficult problem!

Ill-defined problem:

- What is a community/partition?
- What is a *good* community/partition?

Three basic questions

1) How to detect communities?
2) How to test community detection algorithms?
3) How to make partitions robust?

Acknowledgements

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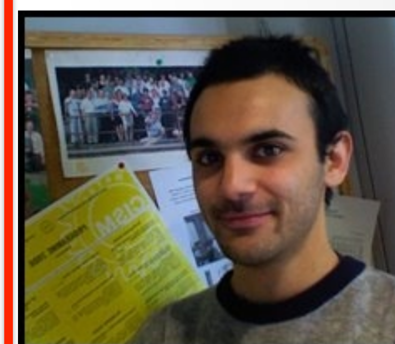
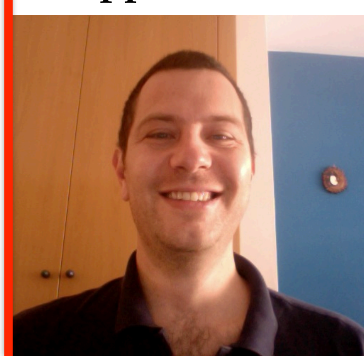
Andrea Lancichinetti



Vito Latora

Massimo Marchiori

Filippo Radicchi



José J. Ramasco

Jari Saramäki

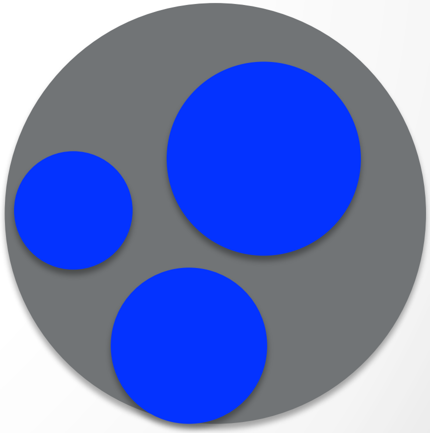
How to detect communities?

Global optimization

Principle:

- Function Q($\mathcal{P}$) that assigns a score to each partition
- Best partition of the network -> partition corresponding to the maximum/minimum of Q($\mathcal{P}$)

Problem: Answer depends on the whole graph -> it changes if one considers portions of it or if it is incomplete



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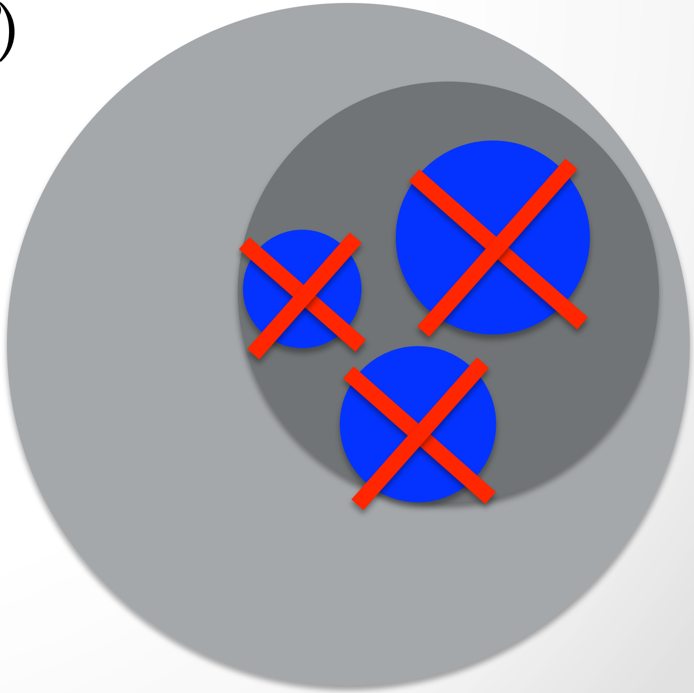

Modularity optimization

$$Q = \frac{1}{m} \sum_{c=1}^{n_c} \left(l_c - \frac{d_c^2}{4m} \right)$$

$$E = m\,c^2$$

M. E. J. Newman, M. Girvan, Phys. Rev. E 69, 026113 (2004)

M. E. J. Newman, Phys. Rev. E 69, 066133 (2004)

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M. E. J. Newman, M. Girvan, Phys. Rev. E 69, 026113 (2004)

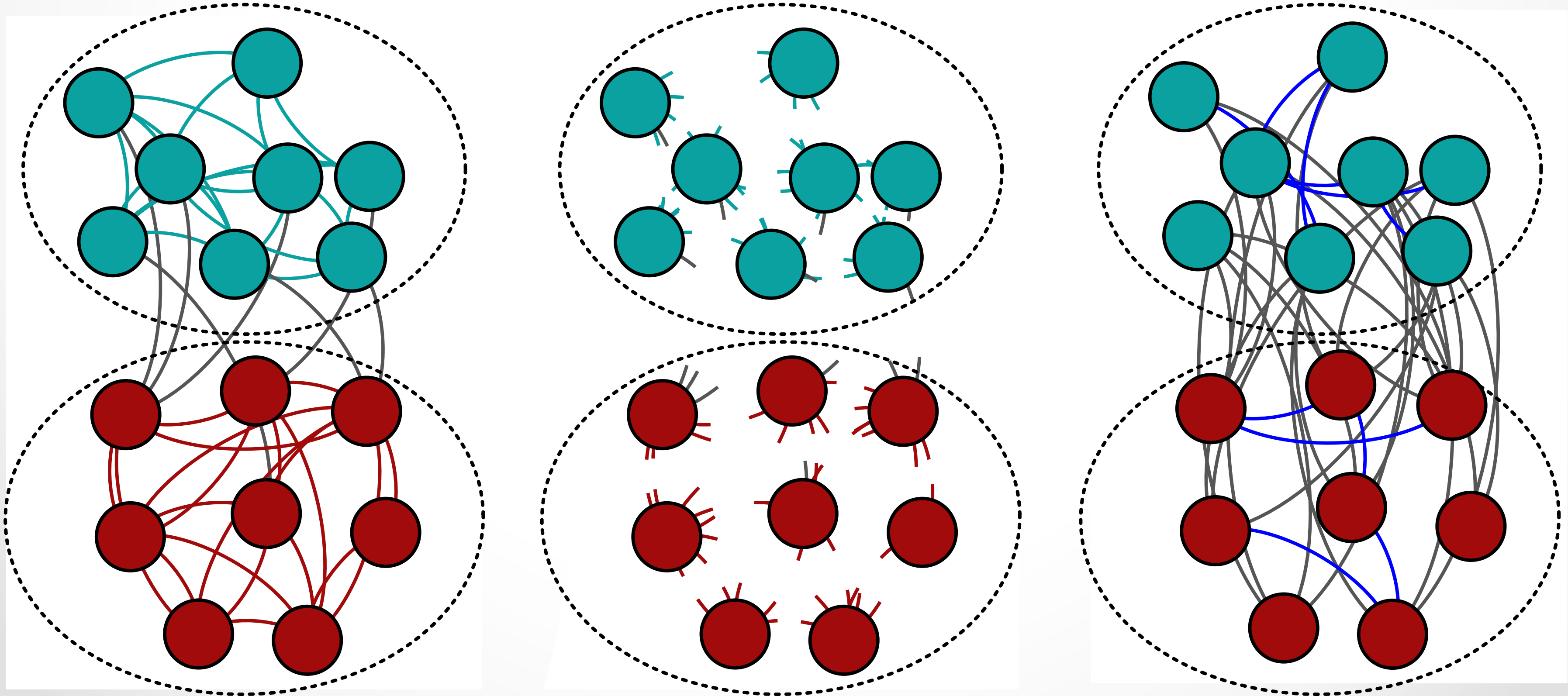
M. E. J. Newman, Phys. Rev. E 69, 066133 (2004)

Goal: find the maximum of Q over all possible network partitions

Problem: NP-complete (Brandes et al., 2007)!

Modularity optimization

$$Q = \frac{1}{m} \sum_{c=1}^{n_c} \left(l_c - \frac{d_c^2}{4m} \right)$$



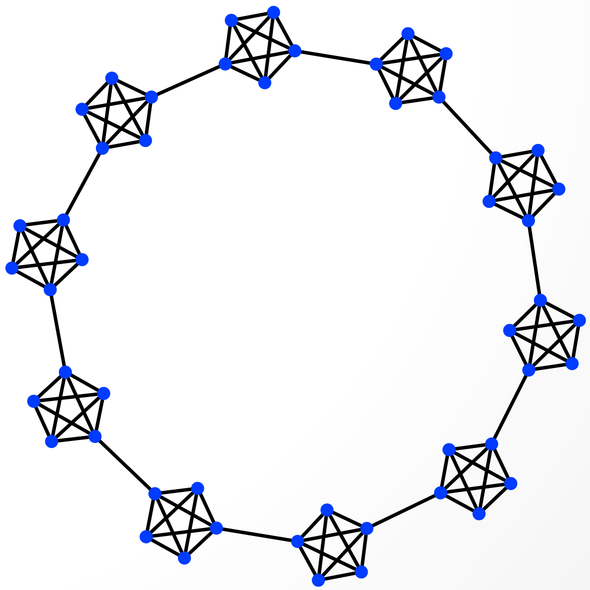
Resolution limit

$$Q = \frac{1}{m}\sum_{c=1}^{n_c}\left[l_c - \frac{1}{4}\left(\frac{d_c}{\sqrt{m}}\right)^2\right]$$

modularity's scale

Result: clusters smaller than this scale cannot be resolved!

Consequences



S. F. & M. Barthélemy, PNAS 104, 36-41 (2007)

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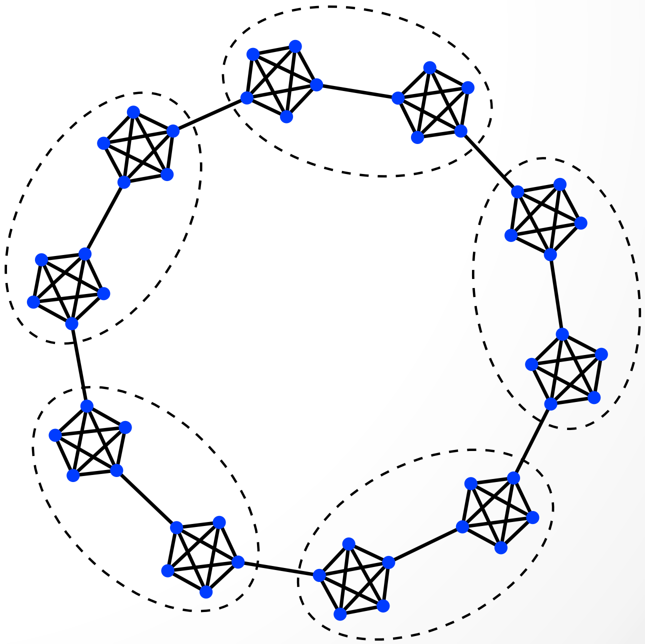

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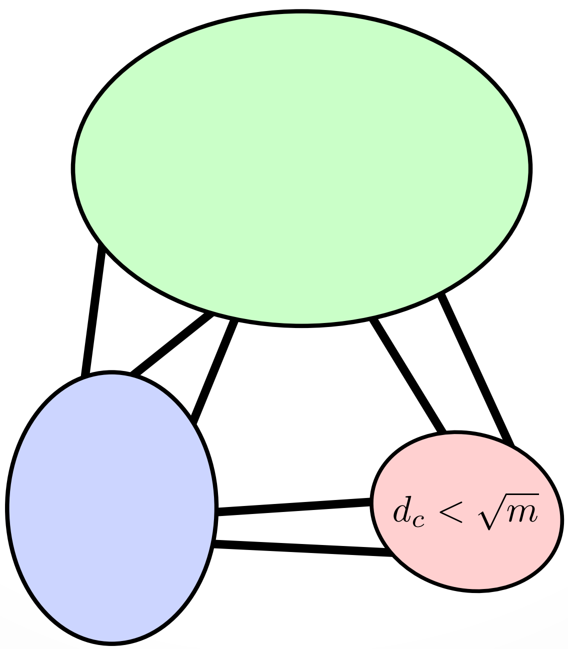

S. F. & M. Barthélemy, PNAS 104, 36-41 (2007)

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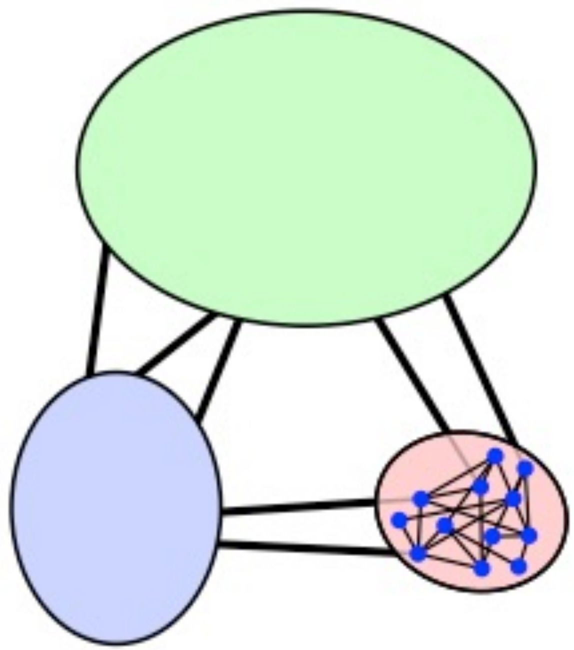

S. F. & M. Barthélemy, PNAS 104, 36-41 (2007)

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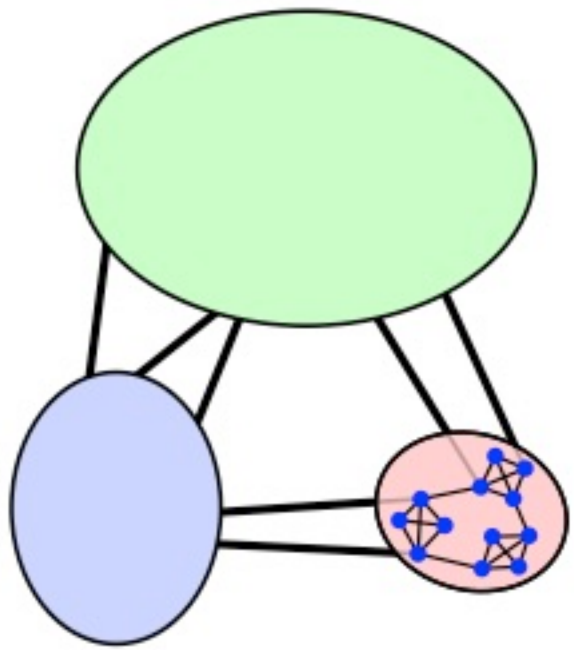
modularity's scale

Consequences



S. F. & M. Barthélemy, PNAS 104, 36-41 (2007)

Local optimization

Principle:

- Communities are local structures
- Local exploration of the network, involving the subgraph and its neighborhood

Advantages:

- Absence of global scales -> no resolution limit
- One can analyze only parts of the network

Local optimization

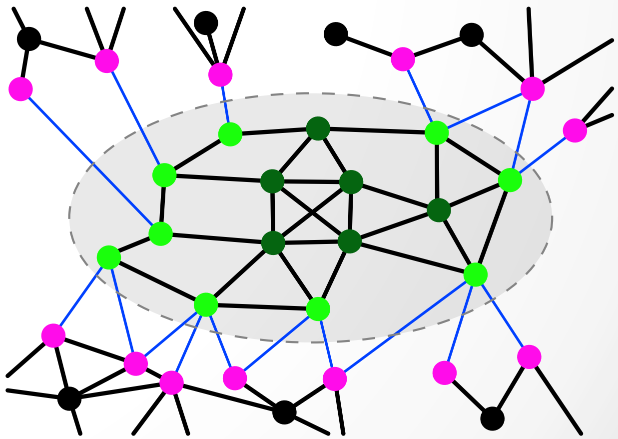
Implementation:

- Function Q(*C*) that assigns a score to each subgraph
- Best cluster -> cluster corresponding to the maximum/minimum of Q(*C*) over the set of subgraphs including a seed node

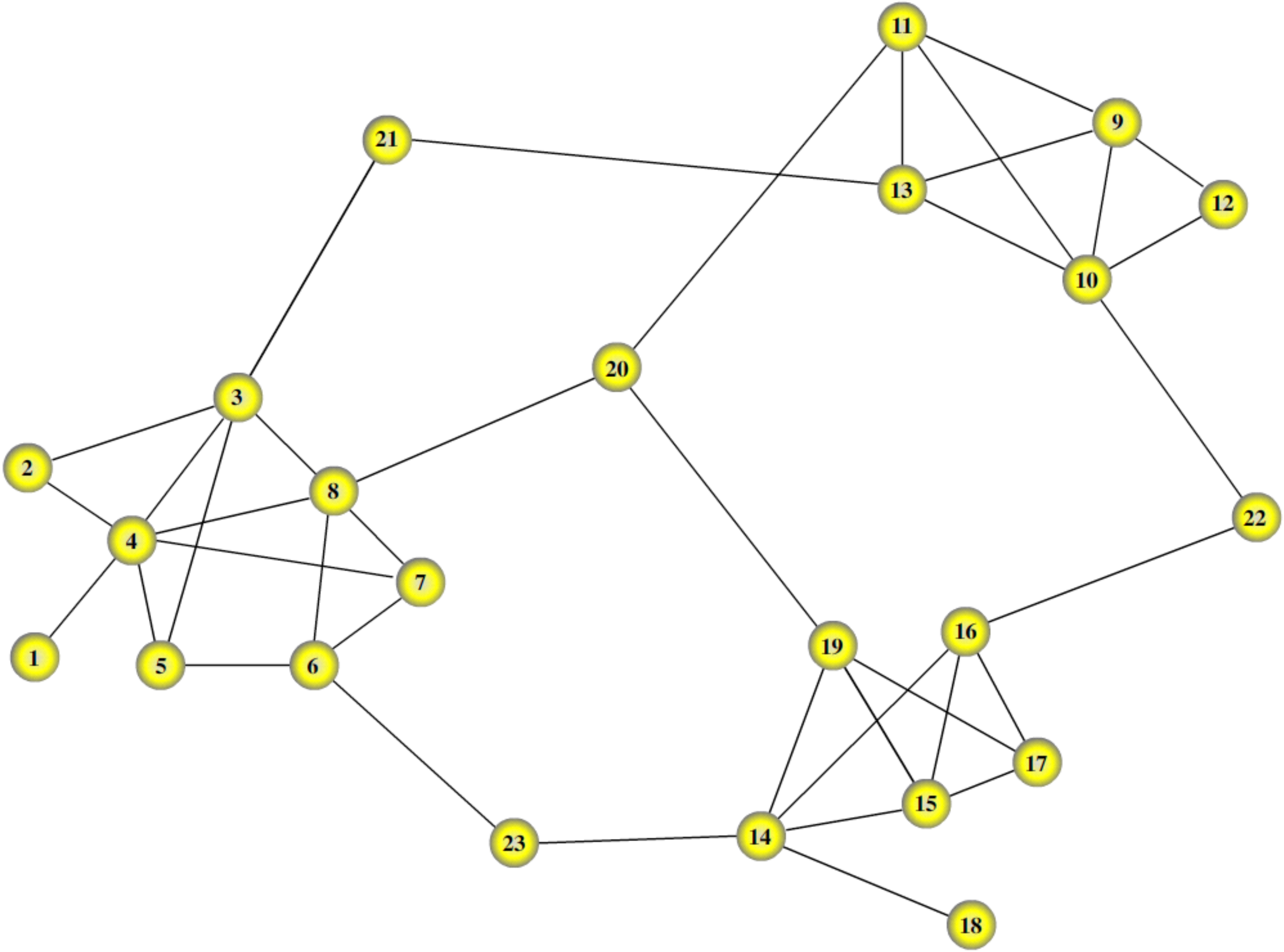
Example: Local Fitness Method (LFM)

Fitness of cluster *C* :

$$f_{\mathcal{C}} = \frac{k_{in}^{\mathcal{C}}}{(k_{in}^{\mathcal{C}} + k_{out}^{\mathcal{C}})^{\alpha}} = \frac{2l_{\mathcal{C}}}{d_{\mathcal{C}}^{\alpha}}$$



A. Lancichinetti, S. F., J. Kertész, New. J. Phys. 11, 033015 (2009)



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Local optimization: OSLOM

Basics:

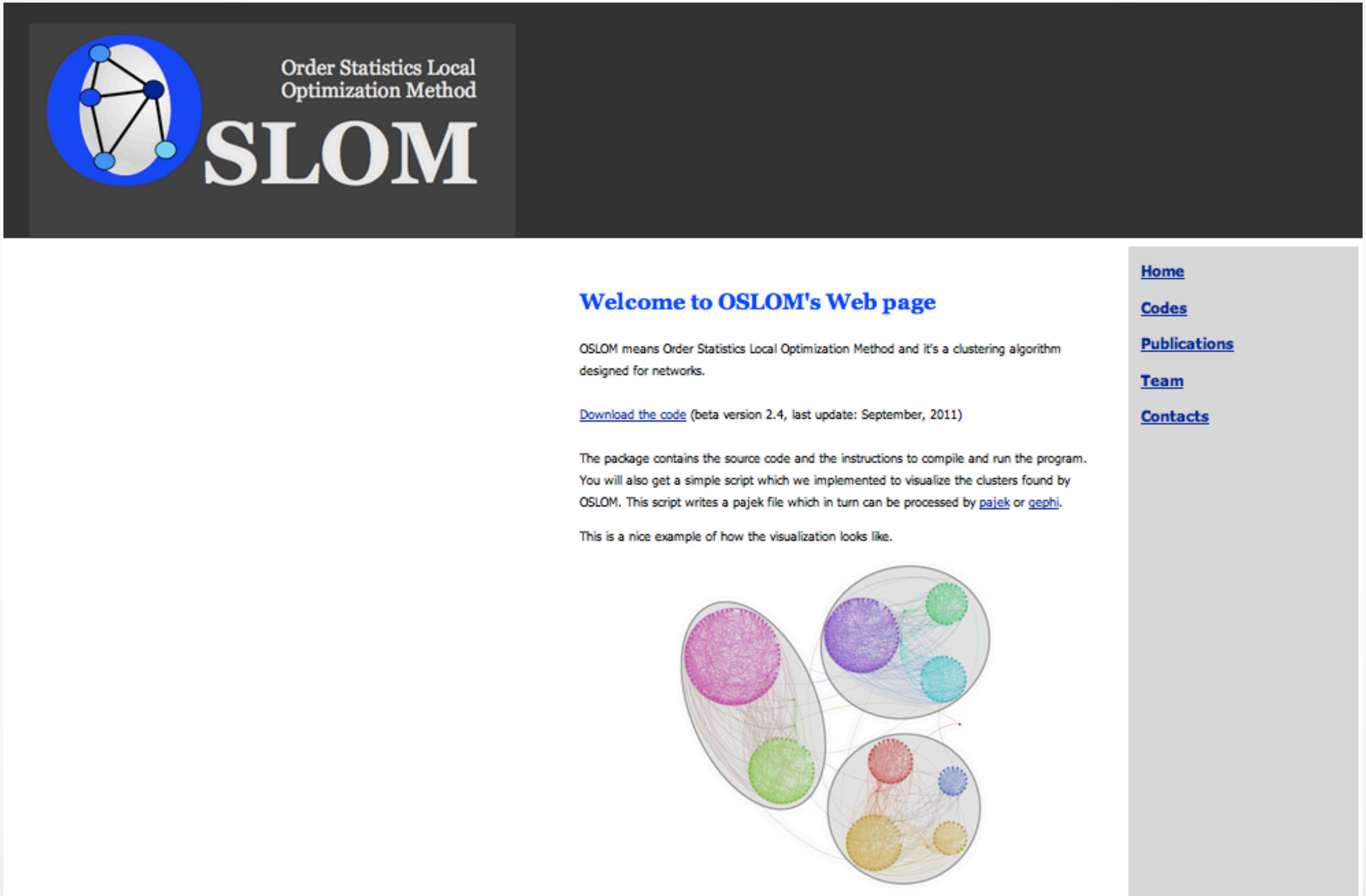
- LFM with fitness expressing the statistical significance of a cluster with respect to random fluctuations
- Statistical significance evaluated with Order Statistics

First multifunctional method:

- Link direction
- Link weight
- Overlapping clusters
- Hierarchy

A. Lancichinetti, F. Radicchi, J. J. Ramasco, S. F., PLoS One 6, e18961 (2011)

Local optimization: OSLOM



http://www.oslom.org/

How to test community detection algorithms?

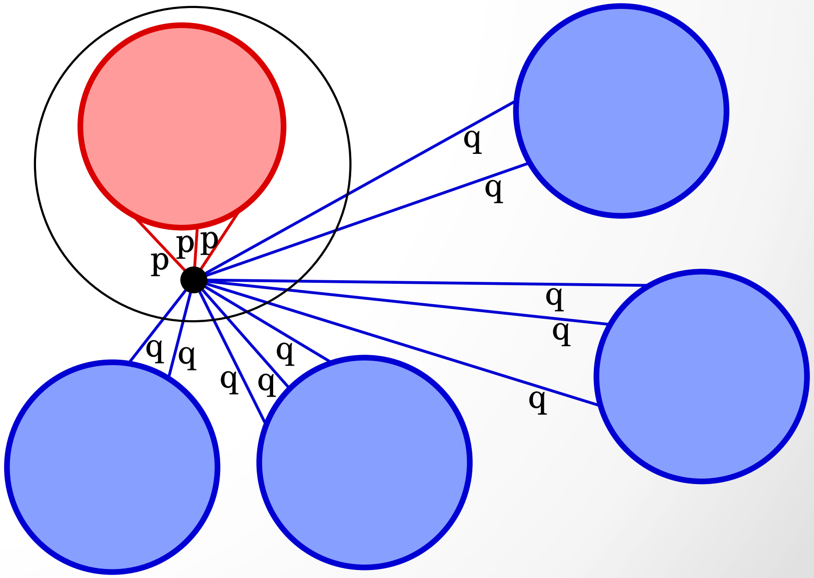
Question: how to test clustering algorithms?

Answer: checking whether they are able to recover the known community structure of benchmark graphs

Planted l-partition model (Condon & Karp, 1999)

Ingredients:

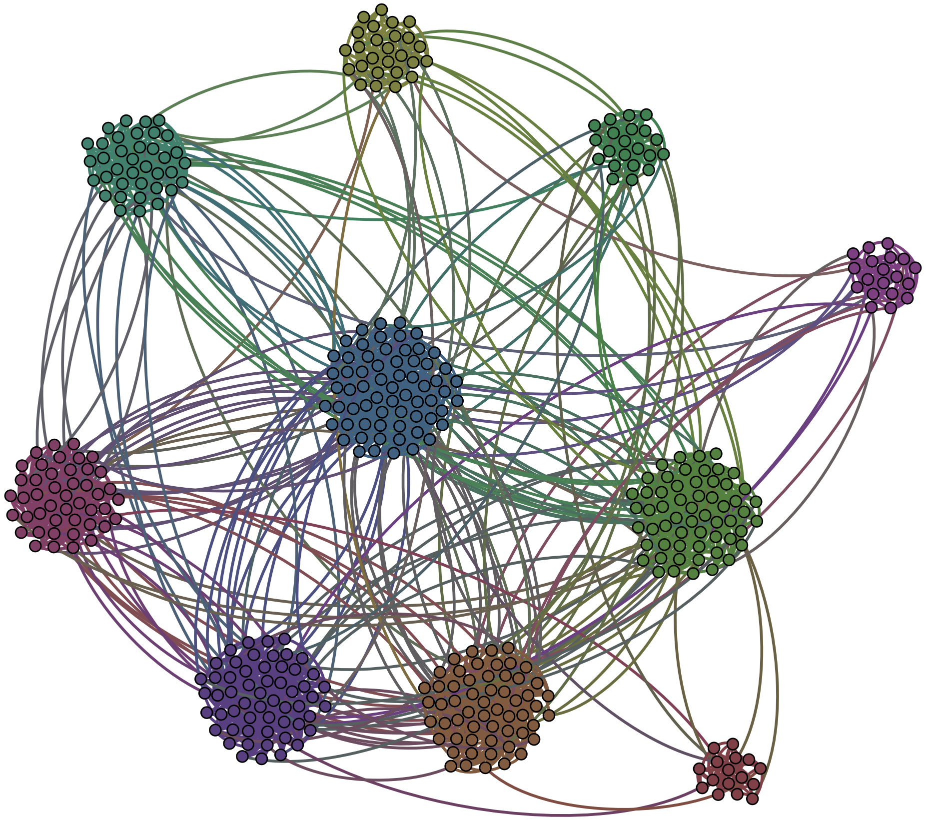
1) p=probability that vertices of the same cluster are joined
2) q=probability that vertices of different clusters are joined



Principle: if p > q the groups are communities

The LFR benchmark

Realistic feature: power law distributions of degree and community size



A. Lancichinetti, S. F., F. Radicchi, Phys. Rev. E 78, 046110 (2008)

https://sites.google.com/site/andrealancichinetti/files/

https://github.com/networkx/networkx/blob/master/networkx/algorithms/community/community_generators.py

The LFR benchmark

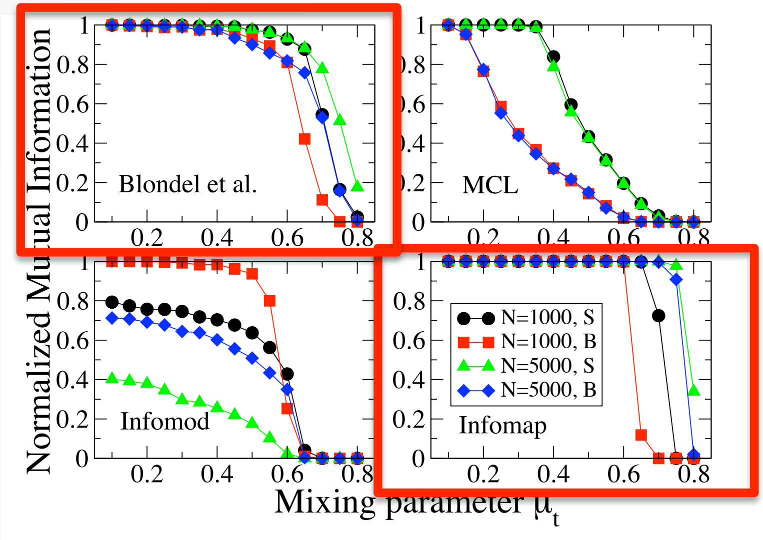
A comparative analysis

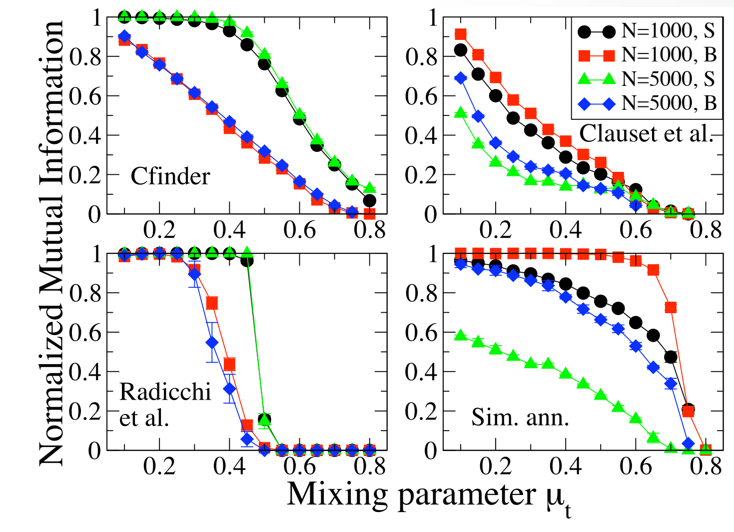
Author	Label	Order
Girvan & Newman	GN	$O(nm^2)$
Clauset et al.	Clauset et al.	$O(n \log^2 n)$
Blondel et al.	Blondel et al.	$O(m)$
Guimerà et al.	Sim. Ann.	parameter dependent
Radicchi et al.	Radicchi et al.	$O(m^4/n^2)$
Palla et al.	Cfinder	$O(\exp(n))$
Van Dongen	MCL	$O(nk^2)$, $k < n$ parameter
Rosvall & Bergstrom	Infomod	parameter dependent
Rosvall & Bergstrom	Infomap	$O(m)$
Donetti & Muñoz	DM	$O(n^3)$
Newman & Leicht	EM	parameter dependent
Ronhovde & Nussinov	RN	$O(n^\beta)$, $\beta \sim 1$

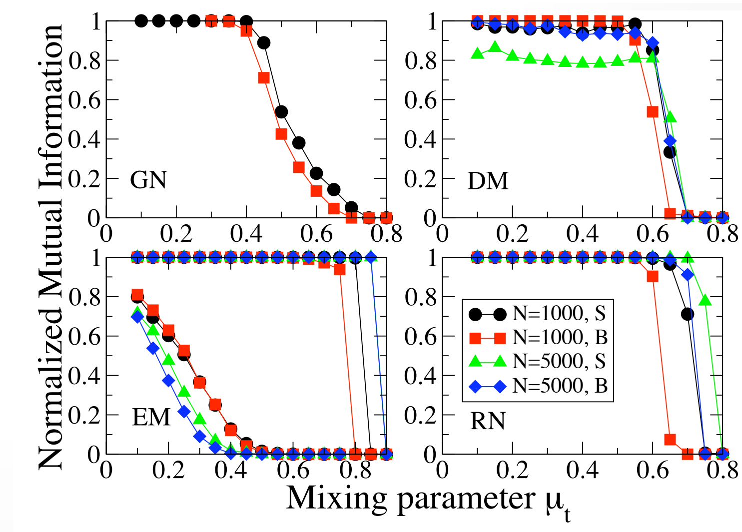
A. Lancichinetti, S. F., Phys. Rev. E 80, 056117 (2009)

The LFR benchmark

A comparative analysis







… and the winner is:

- **Infomap**
- **Louvain method ***

Consensus clustering

Problem: Stochastic (non-deterministic) methods yield many result partitions: which one shall one choose?

Solution: Searching for the partition which is most similar, on average, to the input partitions (*median* or *consensus partition*)

Difficult combinatorial optimization task: greedy solution (consensus matrix)

A. Lancichinetti, S. F., Sci. Rep. 2, 336 (2012).

Consensus matrix

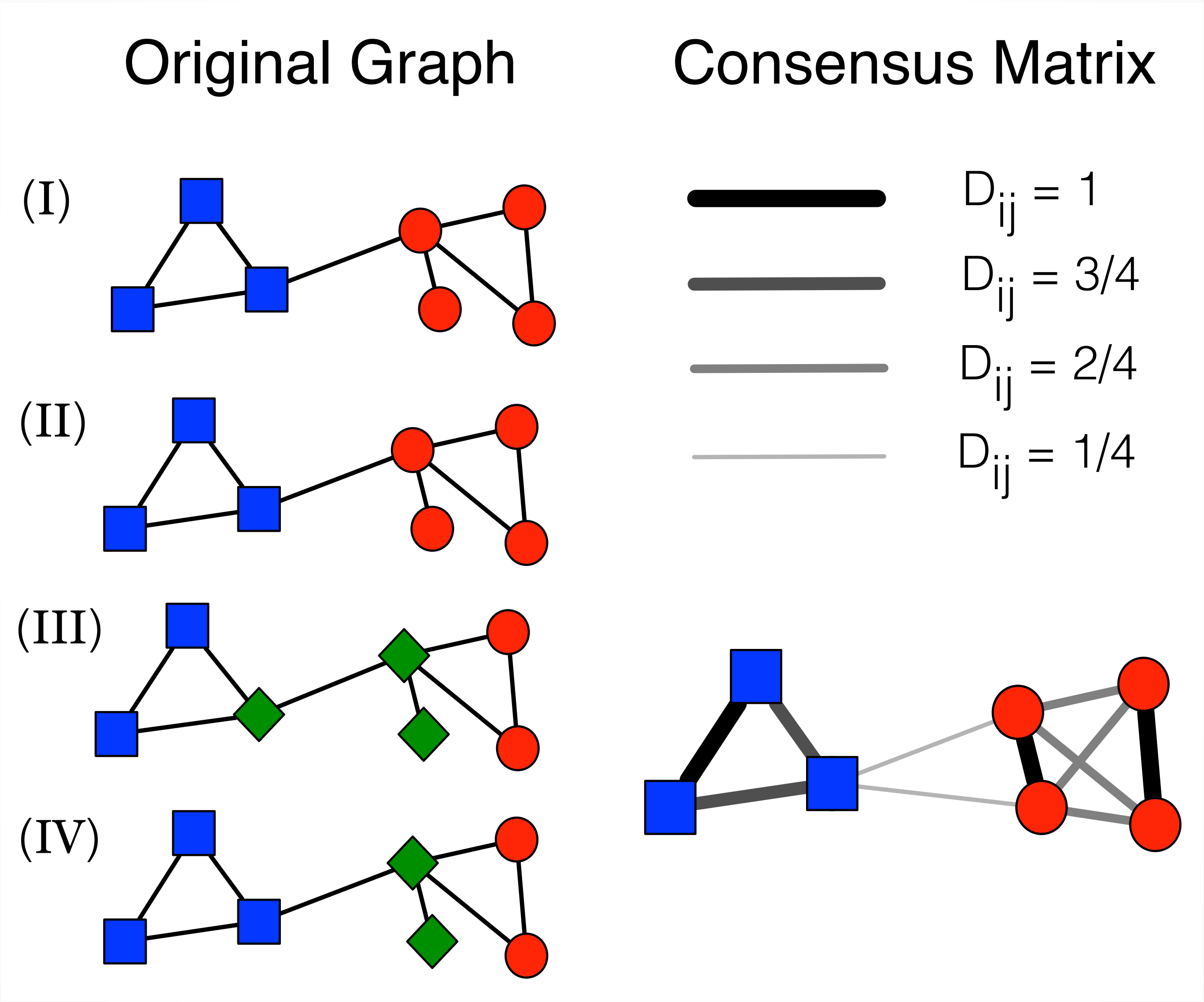
Definition

- Matrix **D** whose entry $\mathbf{D}_{ij}$ is the frequency that vertices i and j were in the same cluster in the input partitions

Starting point: network $\mathcal{G}$ with n vertices, clustering method A.

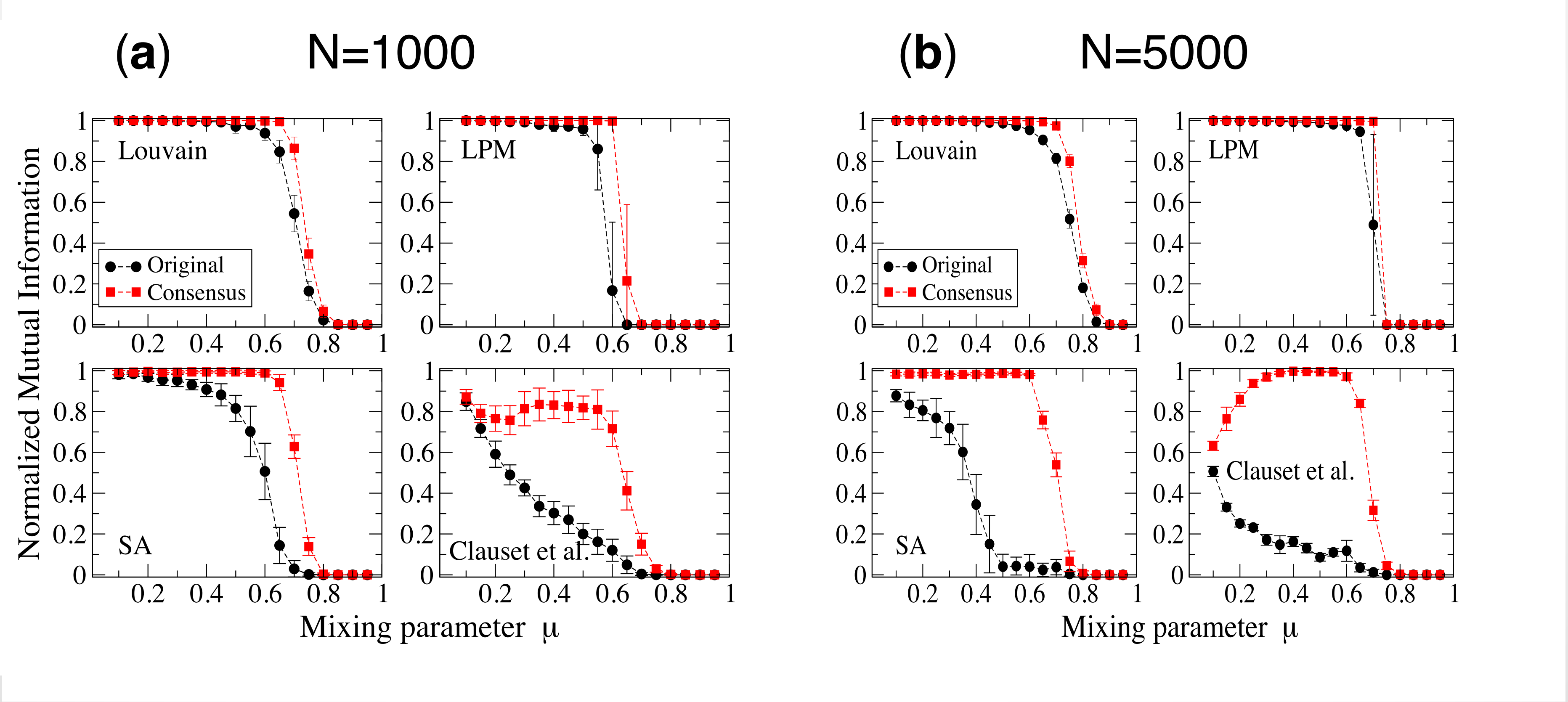
- Apply A on $\mathcal{G}$ n_P times -> n_P partitions
- Compute the consensus matrix **D**: D_{ij} is the number of partitions in which vertices i and j of $\mathcal{G}$ are assigned to the same cluster, divided by n_P
- All entries of **D** below a chosen threshold t are set to zero
- Apply A on **D** n_P times -> n_P partitions
- If the partitions are all equal, stop (the consensus matrix would be block-diagonal). Otherwise go back to 2.

A simple example



Results

LFR benchmarks



Consensus in dynamic networks

- Succession of snapshots, corresponding to overlapping time windows of size Δt: $[t_0, t_0+\Delta t]$, $[t_0+1, t_0+1+\Delta t]$, $[t_m-\Delta t, t_m]$
- D_{ij} = number of times vertices i and j are clustered together, divided by the number of partitions corresponding to snapshots including both vertices

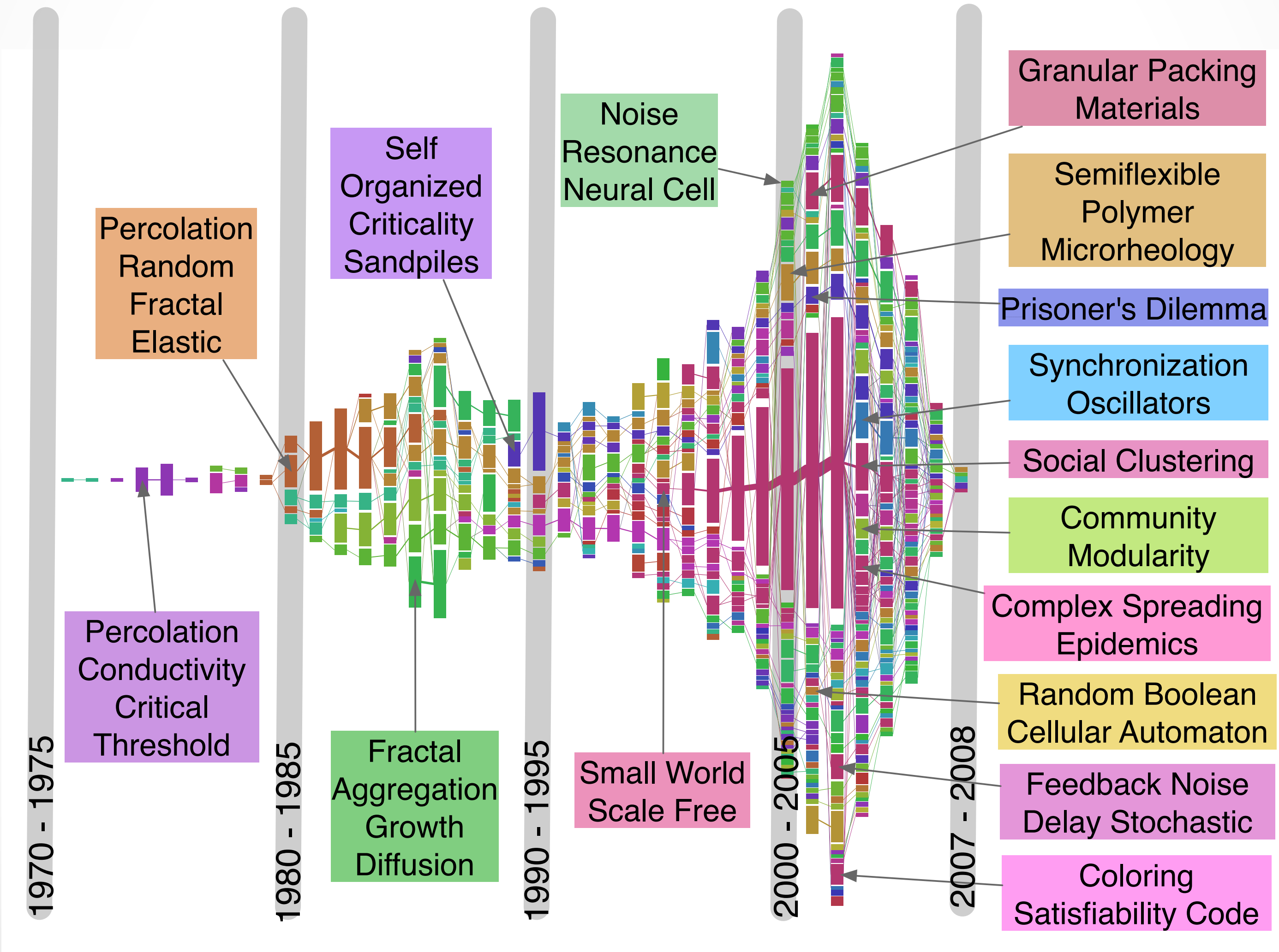
Tracking dynamic clusters: $\mathcal{C}_t \longrightarrow \mathcal{C}_{t+1}$?

Strategy: computing the Jaccard index of C_t with all clusters of the partition at time t + 1, and pick the cluster with the highest value. Same procedure to find the "father" of cluster C_{t+1}

Criterion:

- A and B are each other's best match: A "survives" to time t +1
- A and B are not each other's best match: A "dies" at time t and B is considered as a new cluster.

Tracking dynamic clusters: the APS citation network



Summary

1) What is a community? **No unique answer**! **Definition is system- and problem-dependent**
2) Magic method? **No such thing**! **Domain dependent methods?**
3) **Global optimization** methods have important limits: **local optimization** looks more natural and promising
4) **Consensus clustering** useful technique to find robust partitions
5) Attention on **validation**

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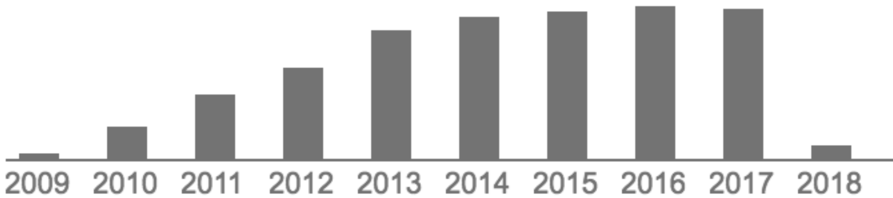

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STRACT

The modern science of networks has brought significant advances to our understanding of complex systems. One of the most relevant features of graphs representing real systems is community structure, or clustering, i.e. the organization of vertices in clusters, with many edges joining vertices of the same cluster and comparatively few edges joining vertices of different clusters. Such clusters, or communities, can be considered as fairly

S. F., Phys. Rep. 486, 75-174 (2010)

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The modern science of networks has brought significant advances to our understanding of complex systems. One of the most relevant features of graphs representing real systems is community structure, or clustering, i.e. the organization of vertices in clusters, with many edges joining vertices of the same cluster and comparatively few edges joining vertices of different clusters. Such clusters, or communities, can be considered as fairly independent compartments of a graph, playing a similar role like, e.g., the tissues or the organs in the human body. Detecting communities is of great importance in sociology, biology and computer science, disciplines where systems are often represented as graphs. This problem is very hard and not yet satisfactorily solved, despite the huge effort of a large interdisciplinary community of scientists working on it over the past few years. We will attempt a thorough exposition of the topic, from the definition of the main elements of the problem, to the presentation of most methods developed, with a special focus on techniques designed by statistical physicists, from the discussion of crucial issues like the significance of clustering and how methods should be tested and compared against each other, to the description of applications to real networks. © 2009 Elsevier B.V.

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Community detection in networks: A user guide

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ABSTRACT

Community detection in networks is one of the most popular topics of modern network science. Communities, or clusters, are usually groups of vertices having higher probability of being connected to each other than to members of other groups, though other patterns are possible. Identifying communities is an ill-defined problem. There are no universal protocols on the fundamental ingredients, like the definition of community itself, nor on other crucial issues, like the validation of algorithms and the comparison of their performances. This has generated a number of confusions and misconceptions, which undermine the progress in the field. We offer a guided tour through the main aspects of the problem. We also point out strengths and weaknesses of popular methods, and give directions to their use.

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Contents

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