Social Network Analysis using Formal Concept Analysis

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Collaboration

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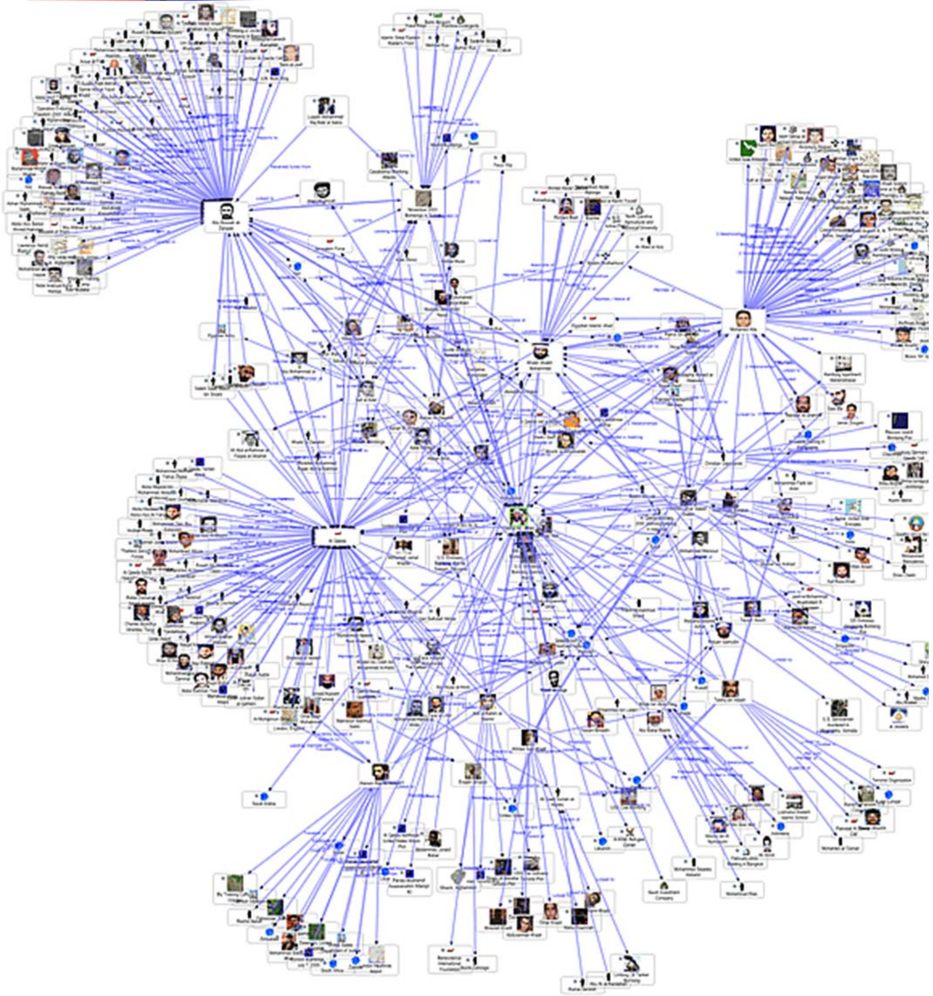
ERIC - Université Lumière Lyon 2
LIMOS - Université Blaise Pascal

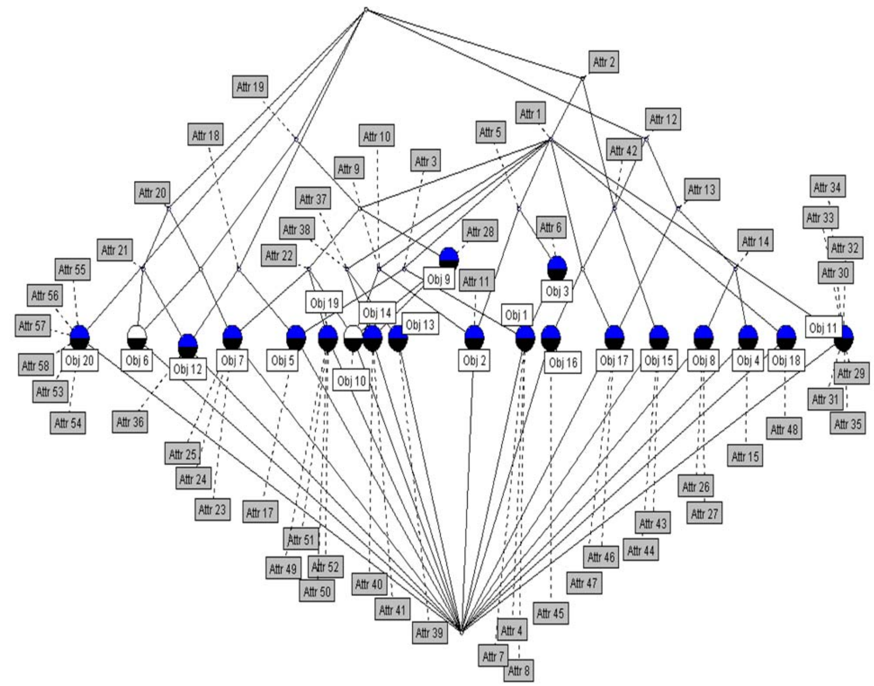
Outline

- Introduction
- Social network analysis (SNA)
- Formal concept analysis (FCA)
- Using FCA for SNA
  - Community and core/peripheral node detection
  - Concept and association rule mining
- Complex structure management
  - Heterogeneous information networks
  - Multimodal data networks
- Conclusion

Introduction



Social network



Concept (Galois) lattice

Introduction

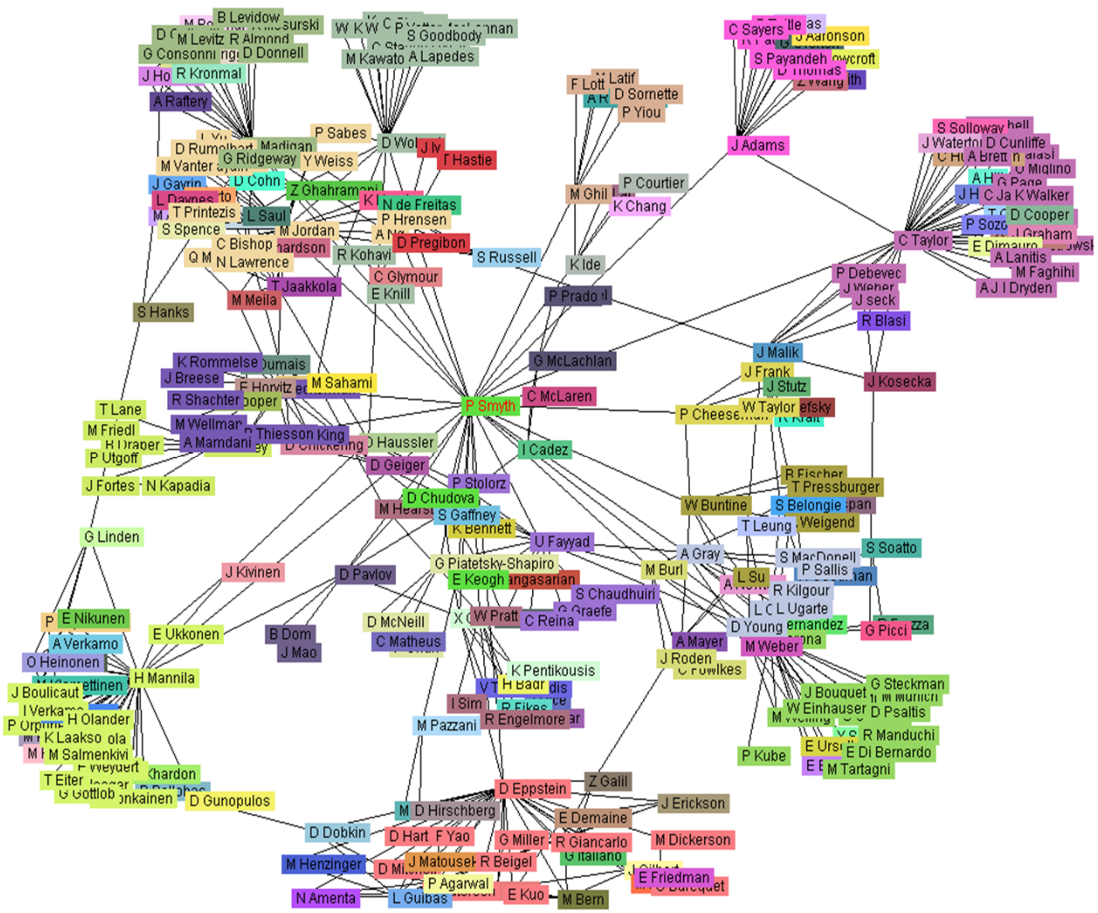
- Big data and complex structures
  - Performance and scalability issues
  - Visualization issues
- Each user has his/her own needs for data analysis
- Data evolution and partitioning
  - Need for incremental algorithms and operations on structures (lattices and graphs)
- Solutions
  - Efficient algorithms and implementations
  - Data selection and decomposition, nested structures
  - Pattern management, browsing, ...

Social Network

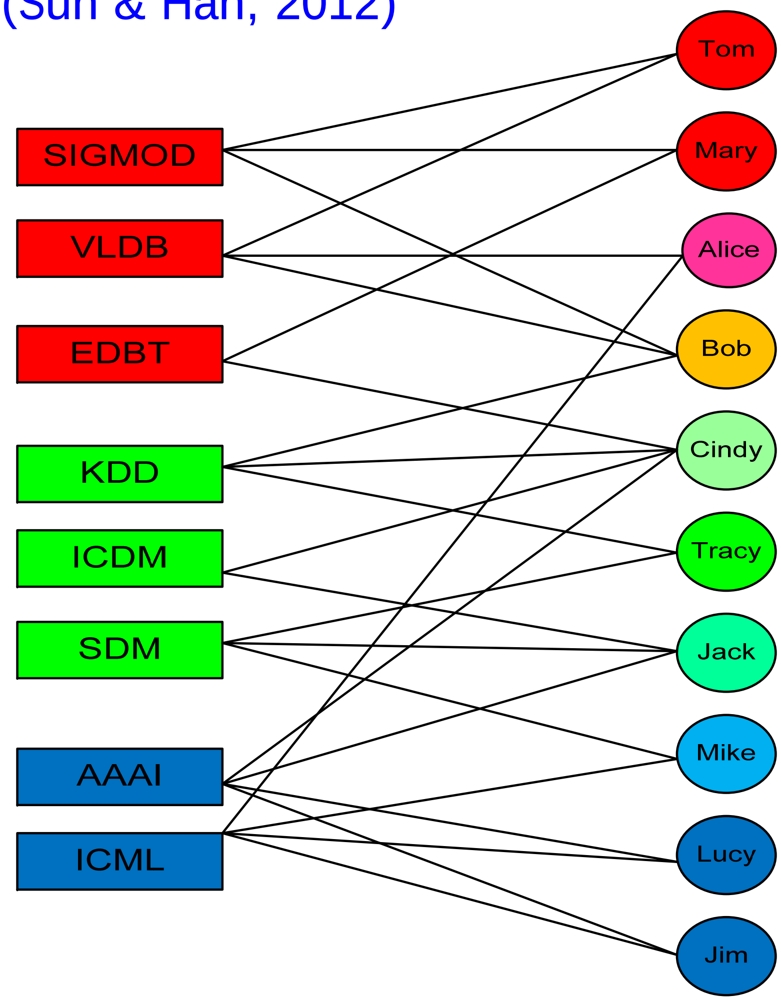
- Definition
  - A social structure of nodes (actors) that are related to each other by various ties such as friendship, affinity, collaboration, …
- Different types of graphs
  - Simple, directed, weighted, or labeled graphs
  - One-mode or many-mode (multidimensional) data
  - Heterogeneous information networks with more than one type of nodes and/or links

One-mode vs. Two-mode Networks

(Sun & Han, 2012)



Co-author Network



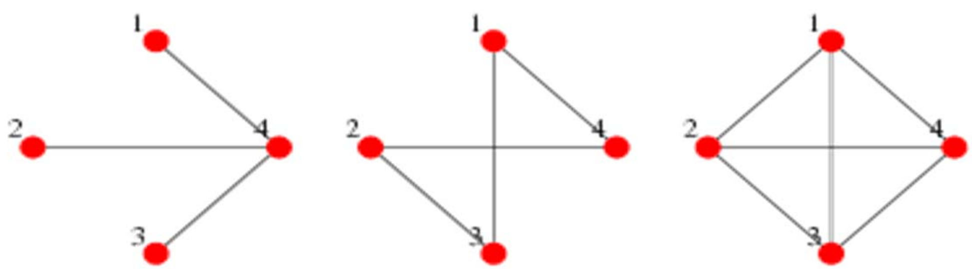
Conference-Author Network

Social Network Analysis

- Many topics
  - Position analysis. E.g., leader or mediator, core/peripheral actor
  - Influence computation and maximization
  - Network reorganization
  - Link prediction and recommendation
  - Community detection and evolution, etc.

One-mode Data

- Interaction networks
  - A graph **G = (V, E)**, where V is a set of vertices/nodes and E a set of edges/links
  - E.g., friendship, co-authorship
- Example



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

http://mathworld.wolfram.com/AdjacencyMatrix.html

One-mode Data

Example. Adjacency matrix

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	1	0	0	0	1	0	0	1	0	0	0	0
3	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0
4	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0
5	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0
6	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0
7	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
8	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0
9	0	0	1	1	1	0	0	1	0	1	0	0	0	0	0
10	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0
11	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0
12	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
13	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
14	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Position Analysis

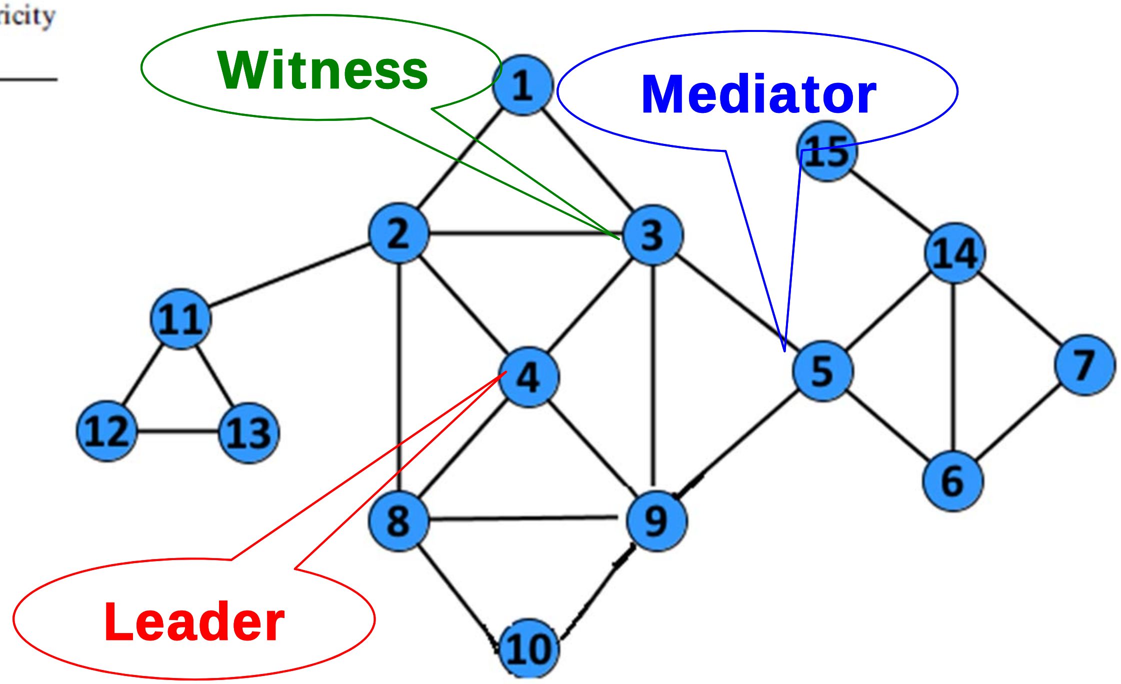
Centrality measure	Interpretation in social networks
Degree	How many people can this person reach directly?
Betweenness	How likely is this person to be the most direct route between two people in the network?
Closeness	How fast can this person reach everyone in the network?
Eigenvector	How well is this person connected to other well-connected people?

CNM Social Media Module – Giorgos Cheliotis (gcheliotis@nus.edu.sg)

An Example

Table 1 The centrality and eccentricity values of the *KITE* nodes

	Degree centrality	Betweenness centrality	Closeness centrality	Eccentricity
1	0.214	0	0.424	4
2	0.357	0.371	0.482	4
3	0.357	0.322	**0.518**	**3**
4	**0.428**	0.067	0.466	4
5	0.285	**0.439**	0.482	4
6	0.214	0.06	0.368	5
7	0.142	0	0.285	6
8	0.285	0.036	0.437	4
9	0.357	0.146	0.466	4
10	0.214	0	0.388	4
11	0.214	0.263	0.368	5
12	0.142	0	0.280	6
13	0.142	0	0.280	6
14	0.285	0.203	0.378	5
15	0.07	0	0.280	6



The eccentricity of a node i is the greatest geodesic distance between i and any other node in the network.

Community Detection

- Find clusters in networks
  - E.g., research communities, Web groups, …
- Methods
  - Hierarchical clustering
  - Girvan–Newman algorithm
  - Modularity maximization
  - Clique based methods (e.g., clique percolation method, Freeman's approach)
  - Biclustering (e.g., block-modeling)
  - Spectral graph partitioning, etc.

Community Detection

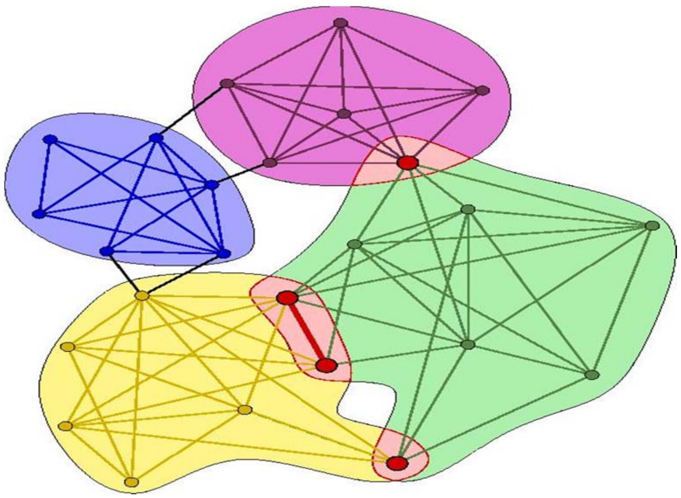
- Algebraic approaches
  - Clique and *n*-cliques
  - Structural and regular equivalence
  - k-cores and k-components
- Algorithmic approaches
  - Larger definition of community: dense connections within a group but sparser ones between groups
  - Partition construction
  - Many algorithms (e.g. modularity maximization, …)

Community Detection

- Cliques and n-cliques in undirected graphs
  - Clique: subgraph of at least three nodes which are all directly connected to one another
  - A maximal clique: a clique which does not exist within a larger one
  - *n*-clique: set of nodes such that the shortest distance between each pair of them is no longer than *n.*

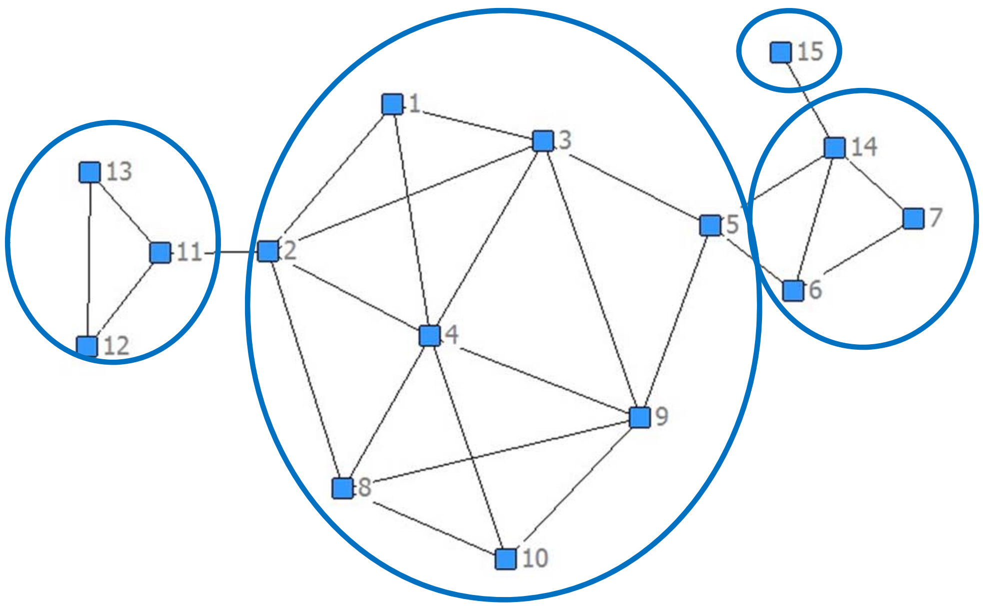
Community Detection

- Algorithmic approaches
  - Agglomerative using for instance similarity measures to produce dendrograms
  - Divisive using e.g. edge betweenness centrality
  - Graph exploration methods such as clique percolation which produces overlapping cliques

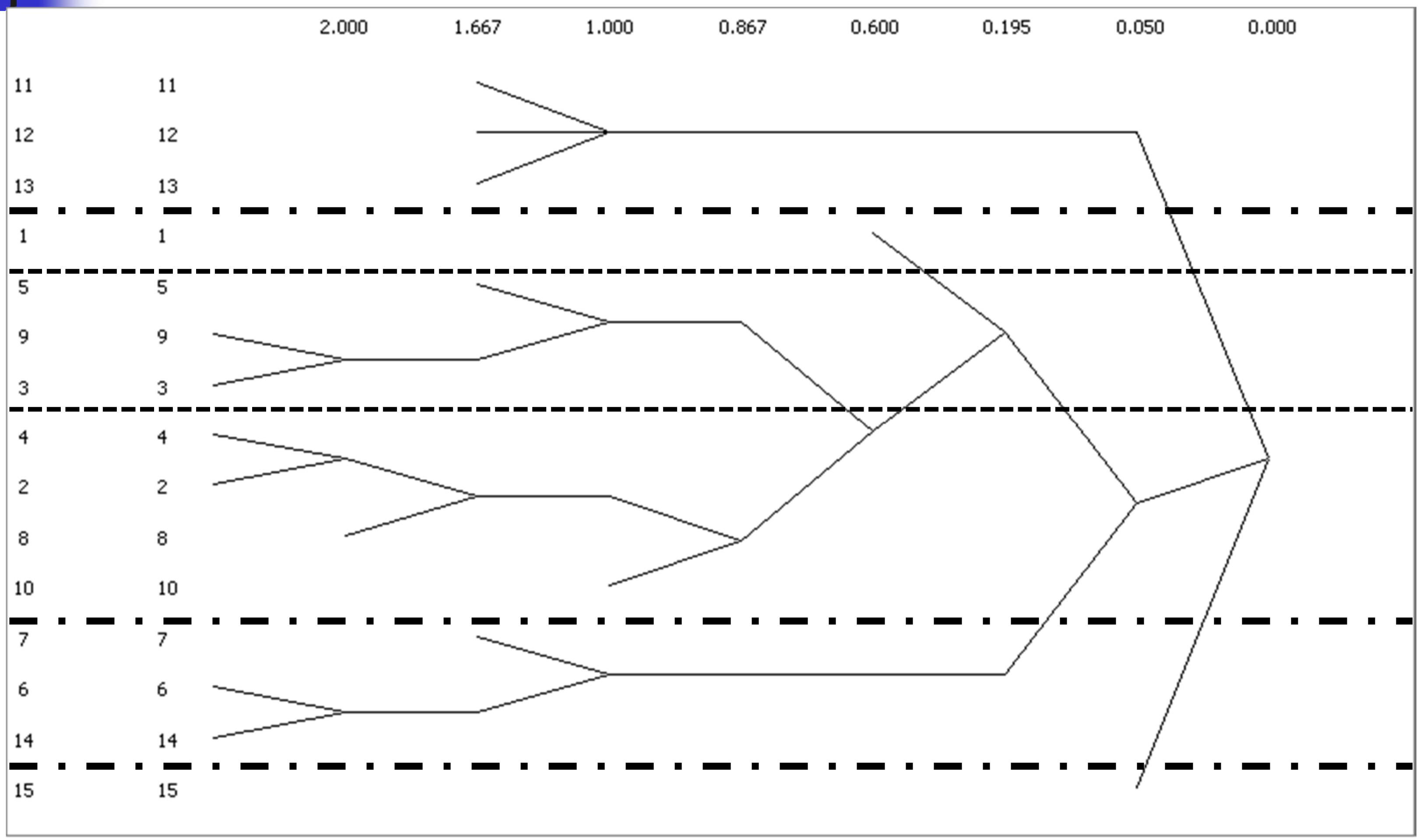


Community Detection

- An example



Community Detection



Dendrogram (produced by UCINET)

Link Prediction

- Objective
  - Predict the link to be created between two nodes, based on the network topology and possibly other features
  - Hard when the network is sparse
- Examples
  - Predict a future link between two Web pages, two researchers, …
- Methods
  - Learning algorithms (e.g., classification) and probabilistic models (e.g., Bayesian networks)
  - Collective prediction, e.g., Markov random field model
  - A proximity-based approach by Liben-Nowell & Kleinberg, etc.

Formal Concept Analysis

- FCA (Ganter & Wille, 1999)
  - Based on lattice and order theory; a conceptual clustering approach; a data mining framework for concepts and association rule computation
- Achievements
  - Efficient algorithms for lattice construction
  - Association rule mining: minimal implication basis, succinct representation of association rules, etc…
  - Extensions to FCA: logical, fuzzy, rough, and relational concept analysis
  - Generalization to *n* dimensions: triadic and polyadic CA
  - Many applications in different domains (SNA, CS, ….)

Formal Concept Analysis

- **Formal context** :=(G, M, I) with $I \subseteq G \times M$.
  $G :\equiv$ set of **objects** and $M :\equiv$ set of **attributes**.
- Derivation. $A \subseteq G$ and $B \subseteq M$.
  $A' := \{m \in M \mid \forall g \in A \quad gIm\}$
  $B' := \{g \in G \mid \forall m \in B \quad gIm\}$.
- **Formal concept** := a pair (A, B) with $A' = B$ and $B' = A$.
  $A :\equiv$ **extent** of (A, B) and $B :\equiv$ **intent** of (A, B).
  $\mathfrak{B}(G, M, I) :=$ set of all concepts of (G, M, I).
- **Concept hierarchy**
  $(A, B) \leq (C, D) :\iff A \subseteq C \qquad (\iff D \subseteq B)$.
- $\underline{\mathfrak{B}}(G, M, I) := (\mathfrak{B}(G, M, I), \leq)$

Formal Concept Analysis

Theorem

$\mathfrak{B}(G, M, I)$ is a complete lattice in which infimum and supremum are given by:

$$\bigwedge_{t \in T} (A_t, B_t) = \left(\bigcap_{t \in T} A_t, \left(\bigcup_{t \in T} B_t \right)'' \right)$$

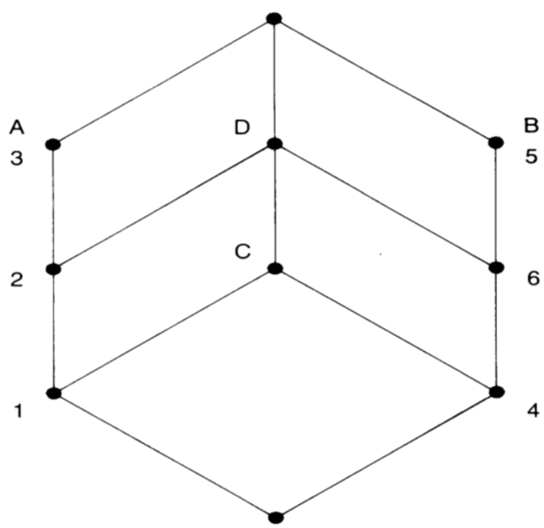
$$\bigvee_{t \in T} (A_t, B_t) = \left(\left(\bigcup_{t \in T} A_t \right)'', \bigcap_{t \in T} B_t \right).$$

$\mathfrak{B}(G, M, I)$ is called the **concept lattice** of the context (G, M, I).

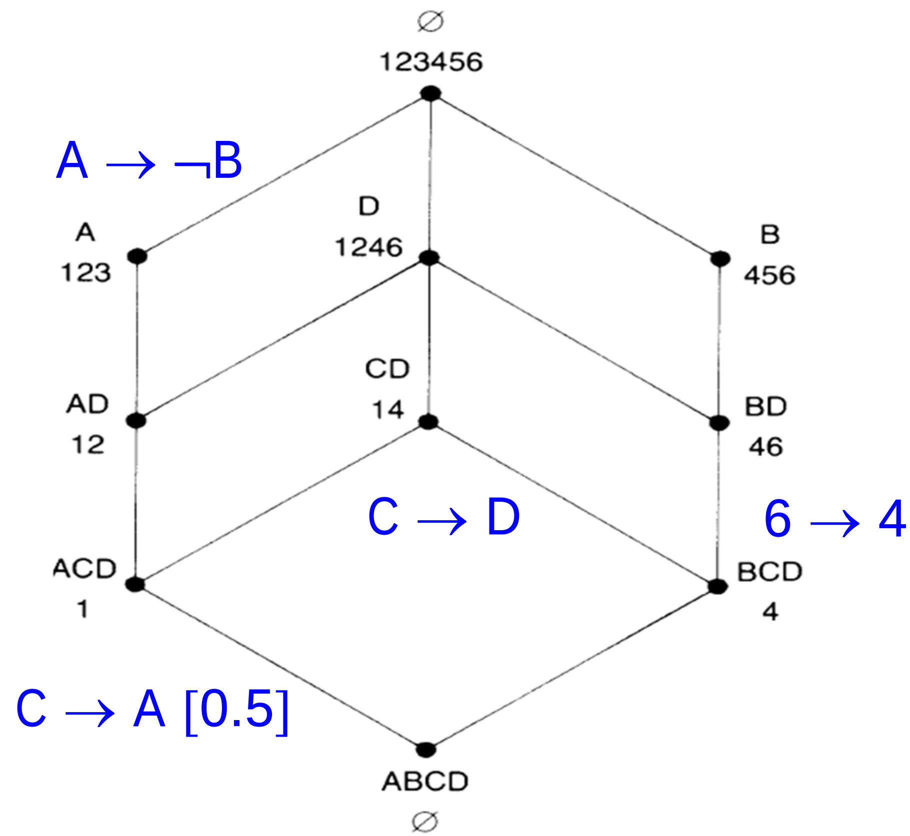
Formal Concept Analysis

Formal context

ACTOR \ EVENT	A	B	C	D
1	1	0	1	1
2	1	0	0	1
3	1	0	0	0
4	0	1	1	1
5	0	1	0	0
6	0	1	0	1

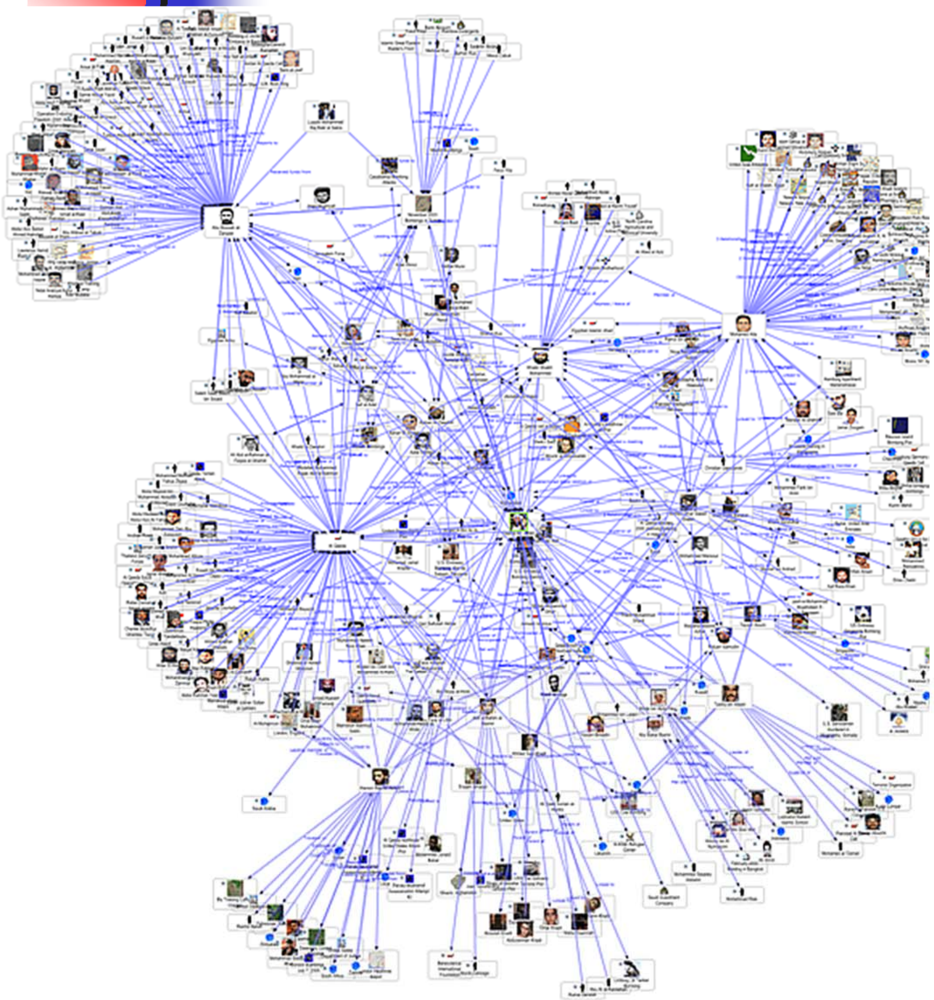


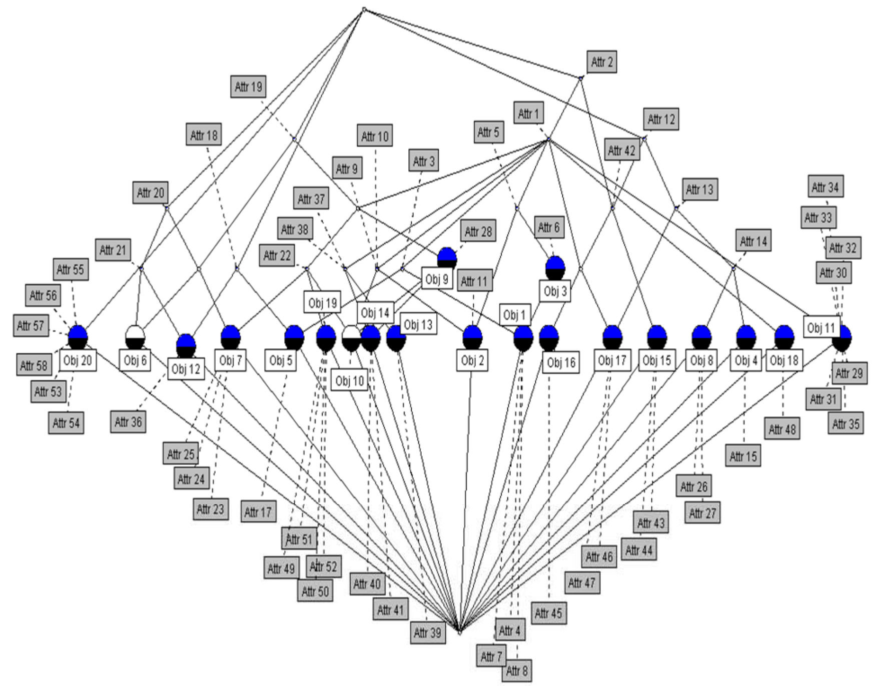
Lattice with reduced labeling



Concept (Galois) lattice

Applying FCA for SNA





Applying FCA for SNA

- Main contributions
  - Analysis of affiliation networks (Freeman & White, 1993)
  - Special issue of Social Networks in 1996
    - E.g., analysis of interaction networks using cliques and FCA (Freeman, 1996)
  - Stability index of a concept (Kuznetsov 2007)
  - Web communities (Rome & Haralick, 2005)
  - Folksonomy analysis (Jäschke et al., 2006)
  - Workshop on SNA using FCA (Obiedkov et al., 2007)
  - Citation analysis (Tilley & Eklund, 2007)
  - FCA in Sociology (Duquenne & Mohr, 2008), etc.

Freeman's Approach to Group Detection

- Extract maximal cliques from a one-mode data network
- Form a formal context where objects are individuals and attributes are maximal cliques
- Construct the concept lattice
- Identify bridging cliques and edges,
- Eliminate bridging edges to produce **communities**
- Central actors are near the bottom of the lattice while peripheral ones are in the upper part

Formal context

Objects are actors and attributes are maximal cliques

	A	B	C	D	E	F	G	H
1	1	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0
3	1	0	0	1	1	0	0	0
4	1	1	1	1	0	0	0	0
5	0	0	0	0	1	1	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	0	1	0
8	0	1	1	0	0	0	0	0
9	0	0	1	1	1	0	0	0
10	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	1
13	0	0	0	0	0	0	0	1
14	0	0	0	0	0	1	1	0
15	0	0	0	0	0	0	0	0

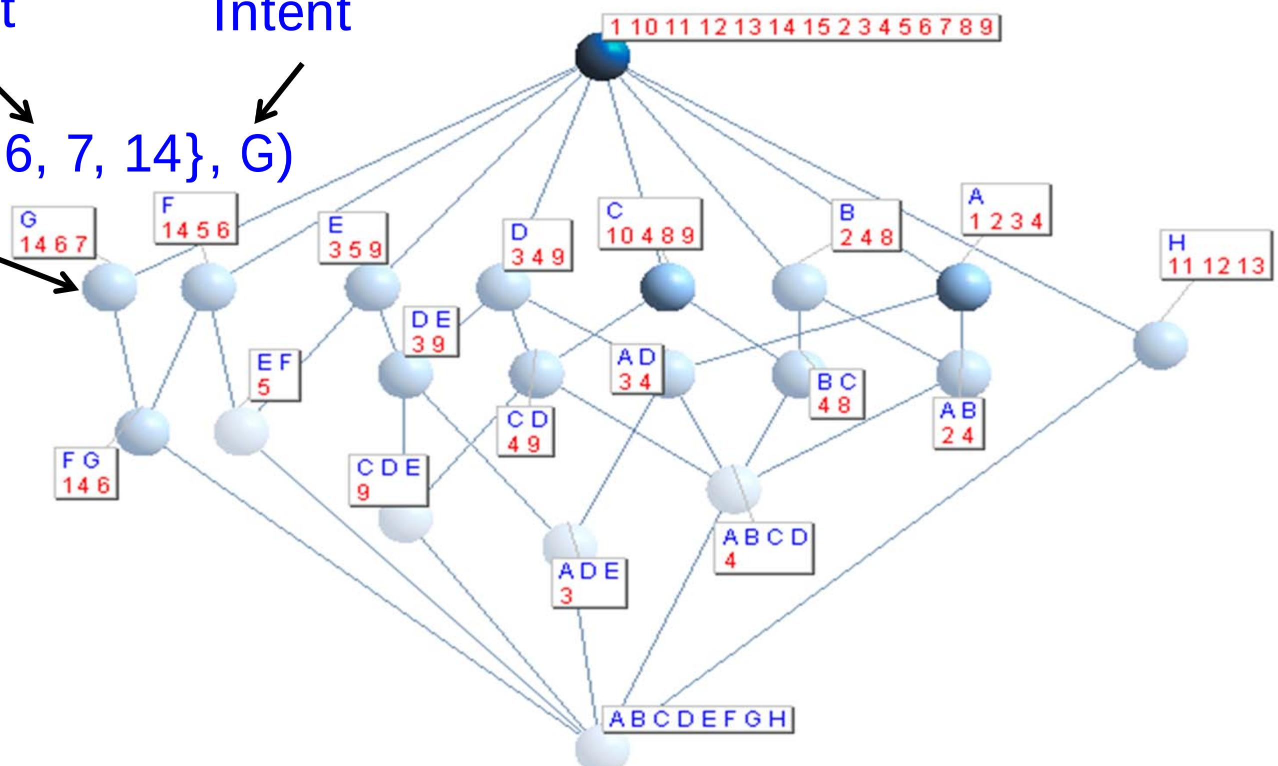
Concept Lattice

Supremum = ({1, ..., 15}, ∅)

Extent

Intent

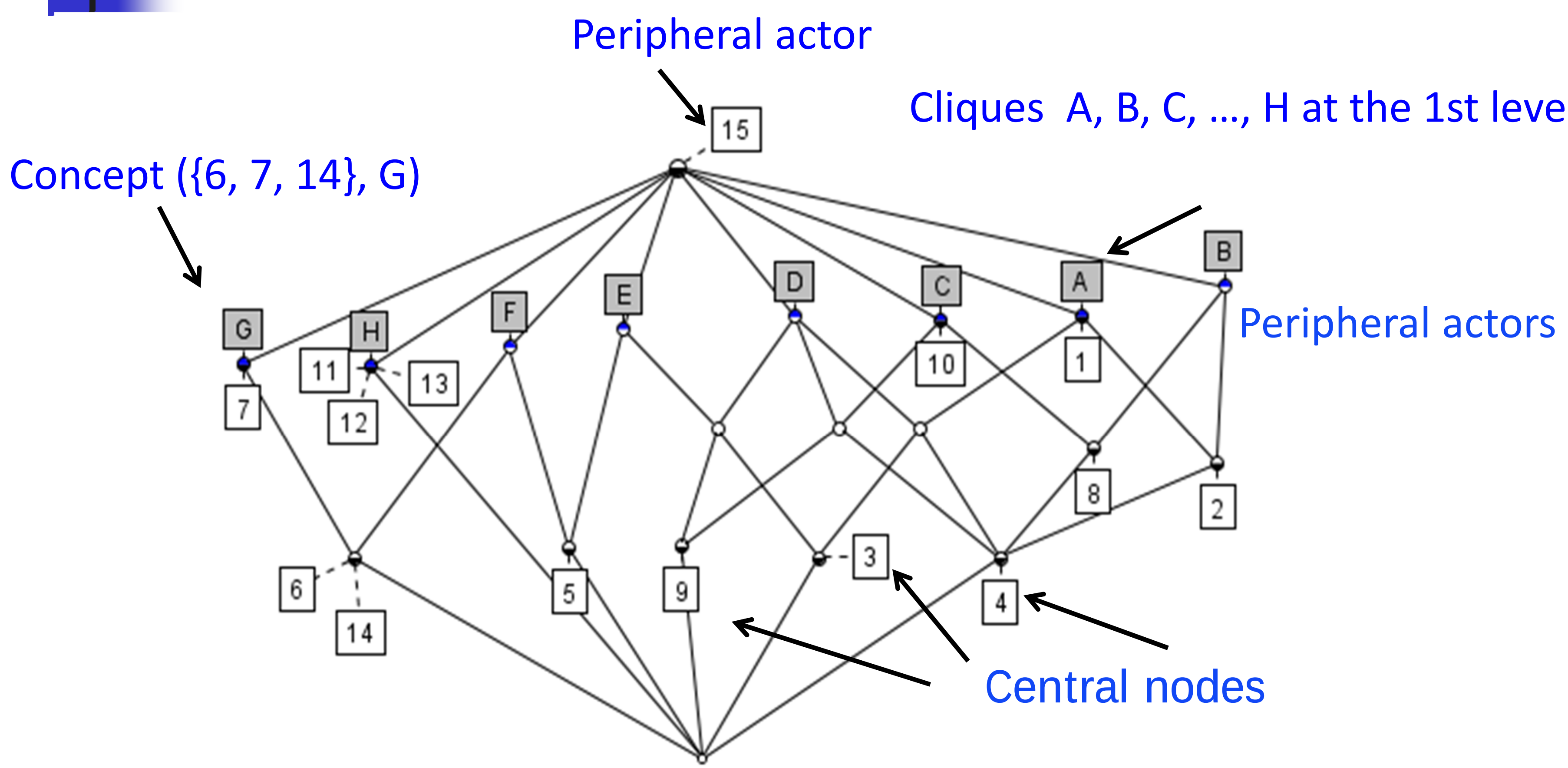
Concept ({6, 7, 14}, G)



Infimum = (∅, {A, ..., H})

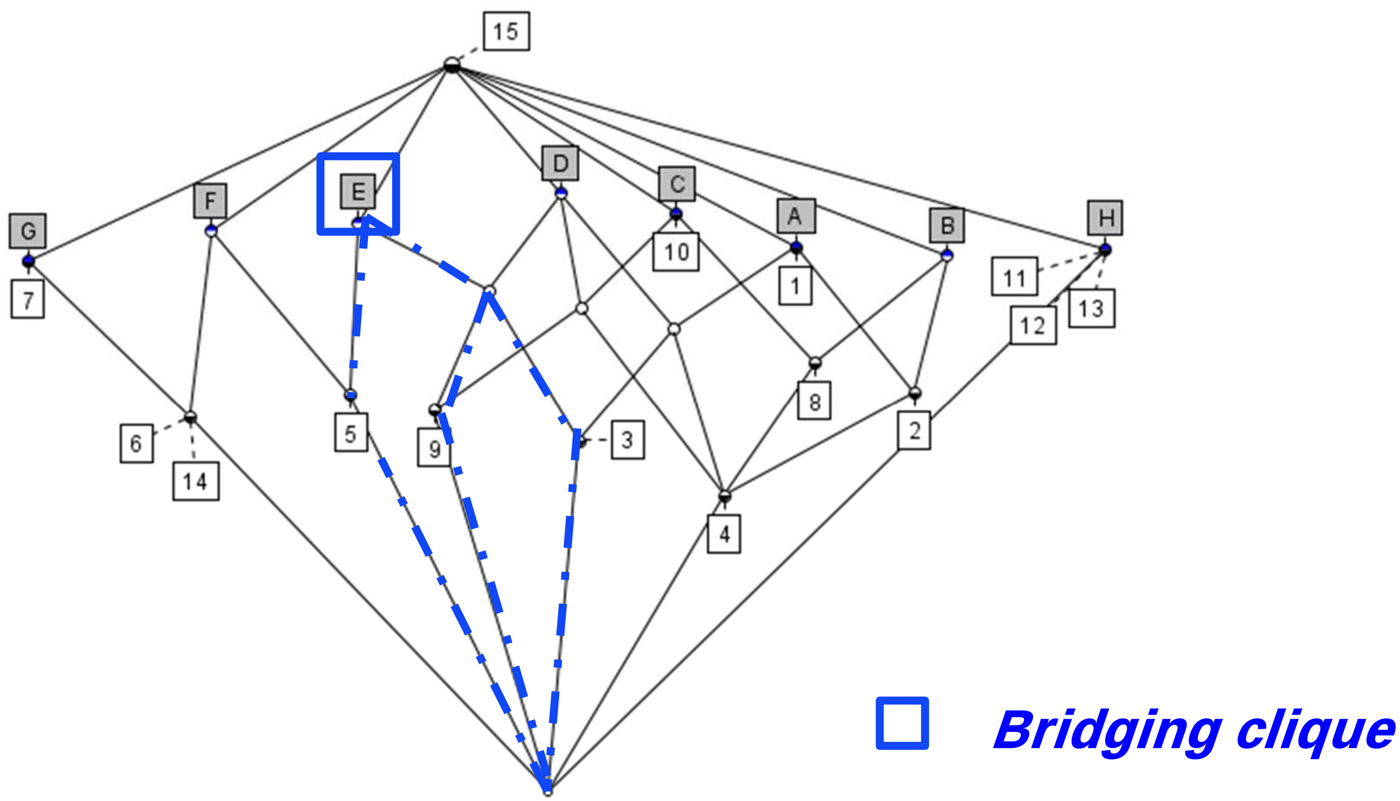
Lattice with full labelling

Concept Lattice



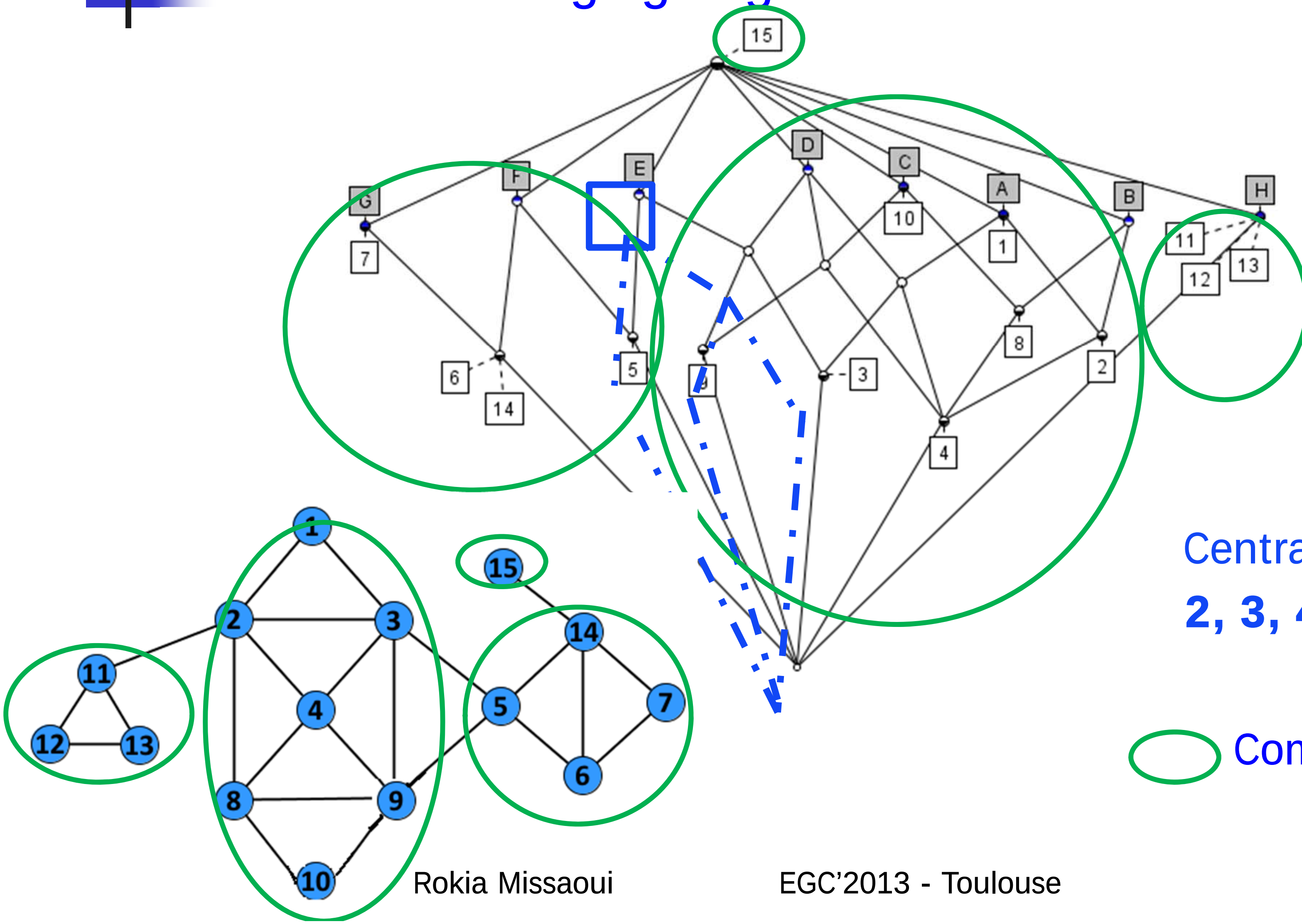
Lattice with reduced labelling

Bridging Cliques & Edges



Community Detection

Deletion of bridging edges



Central nodes

2, 3, 4, 5, **9**, 6, 14

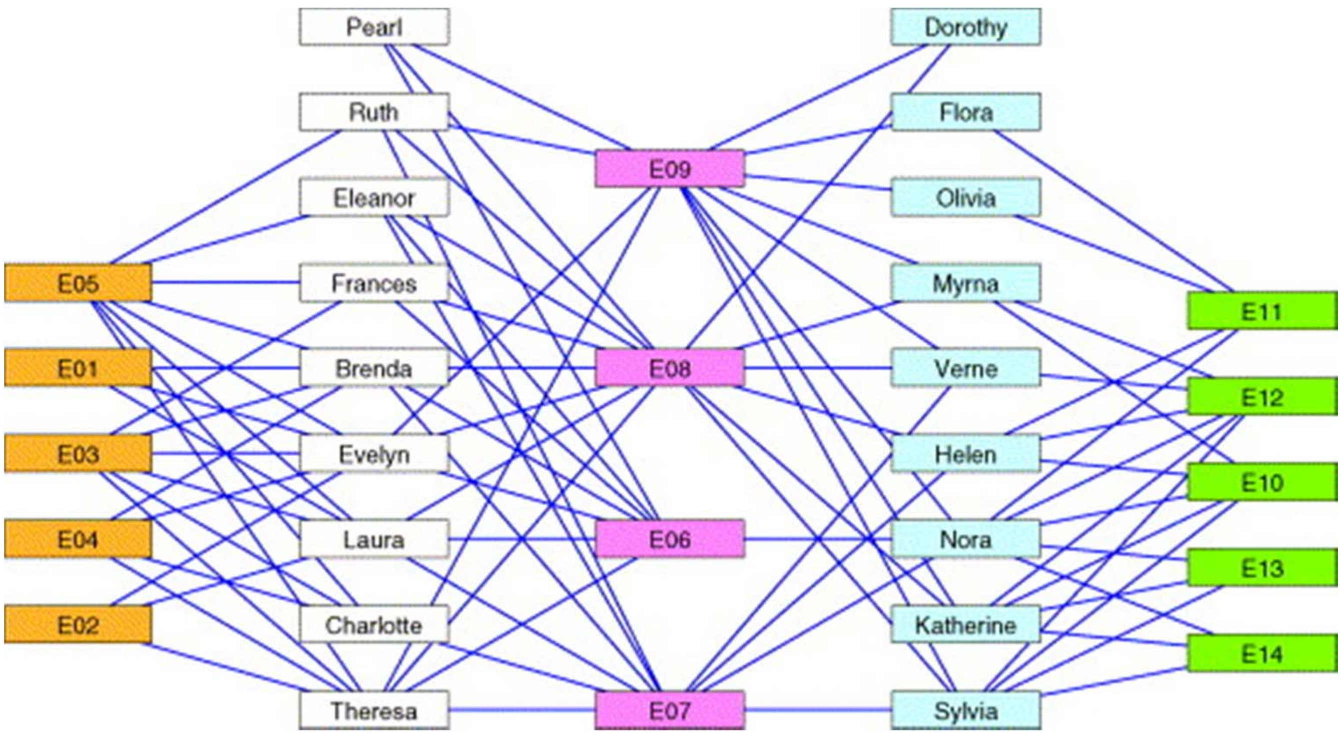
Community

Community Detection

- Limits of Freeman's approach
  - The notion of clique is too restrictive: no cliques $\rightarrow$ no communities!
  - There may be many bridging edges
  - Some nodes (even core ones) are lost after the removal of bridging edges
- Improvement in (Falzon, 2000)
  - All the lattice layers are exploited rather than the clique (first) layer only
  - No node is lost

Two-mode Data

- $G=(V_1 \cup V_2, E \subseteq V_1 \times V_2)$, bipartite graph
- E.g., Southern women attending events



Two-mode Data

- Participation of women to events (Davis)

ACTOR	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	1	1	1	1	1	1	0	1	1	0	0	0	0	0
2	1	1	1	0	1	1	1	1	0	0	0	0	0	0
3	0	1	1	1	1	1	1	1	1	0	0	0	0	0
4	1	0	1	1	1	1	1	1	0	0	0	0	0	0
5	0	0	1	1	1	0	1	0	0	0	0	0	0	0
6	0	0	1	0	1	1	0	1	0	0	0	0	0	0
7	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8	0	0	0	0	0	1	0	1	1	0	0	0	0	0
9	0	0	0	0	1	0	1	1	1	0	0	0	0	0
10	0	0	0	0	0	0	1	1	1	0	0	1	0	0
11	0	0	0	0	0	0	0	1	1	1	0	1	0	0
12	0	0	0	0	0	0	0	1	1	1	0	1	1	1
13	0	0	0	0	0	0	1	1	1	1	0	1	1	1
14	0	0	0	0	0	1	1	0	1	1	1	1	1	1
15	0	0	0	0	0	0	1	1	0	1	1	1	0	0
16	0	0	0	0	0	0	0	1	1	0	0	0	0	0
17	0	0	0	0	0	0	0	0	1	0	1	0	0	0
18	0	0	0	0	0	0	0	0	1	0	1	0	0	0

(Column header: EVENT)

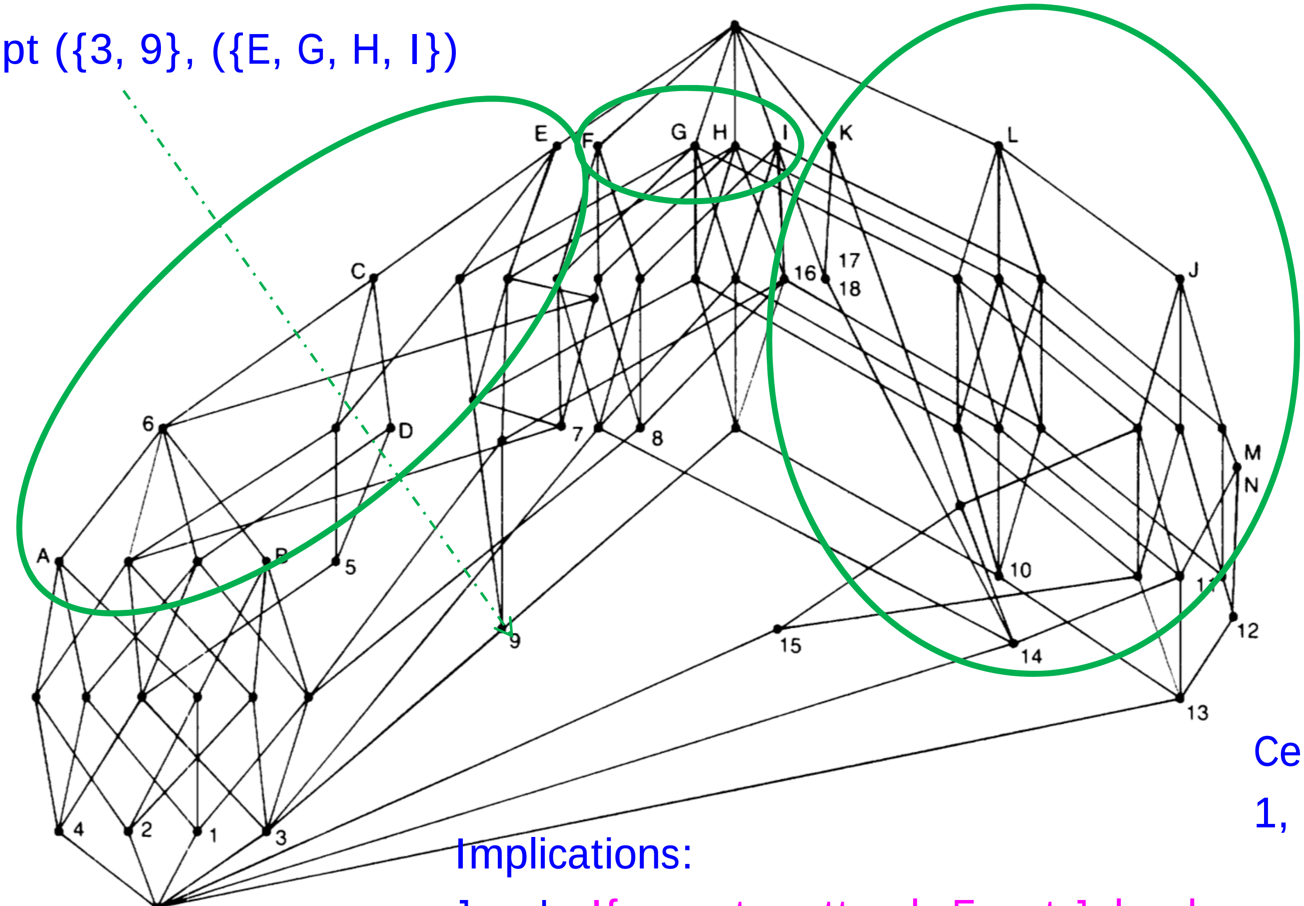
FIGURE 5. Davis, Gardner, and Gardner's two mode data.

Concept Lattice

Actors: 1, .. 18

Events: A, ...N

Concept ({3, 9}, ({E, G, H, I})



Three event groups:

G1= {A, B, ... E}

G2= {F, G, H, I}

G3= {J, K, L, M, N}

Central actors:

1, 2, 3, 4, 12, 13, 14, 15

Implications:

J → L: If an actor attends Event J, he does so for Event L

5 → 3, 4: Events attended by Actor 5 are also attended by 3 & 4

Conversion to one-mode Data

- Projection using matrix multiplication
  - A x A^T gives the number of events co-attended by both the row and the column women

		1 E	2 L	3 T	4 B	5 C	6 F	7 E	8 P	9 R	10 V	11 M	12 K	13 S	14 N	15 H	16 D	17 O	18 F
1	EVELYN	8	6	7	6	3	4	3	3	3	2	2	2	2	2	1	2	1	1
2	LAURA	6	7	[illegible]	6	3	4	4	2	3	2	1	1	2	2	2	1	0	0
3	THERESA	7	6	8	6	4	4	4	3	4	3	2	2	3	3	2	2	1	1
4	BRENDA	6	6	6	7	4	4	4	2	3	2	1	1	2	2	2	1	0	0
5	CHARLOTTE	3	3	4	4	4	2	2	0	2	1	0	0	1	1	1	0	0	0
6	FRANCES	4	4	4	4	2	4	3	2	2	1	1	1	1	1	1	1	0	0
7	ELEANOR	3	4	4	4	2	3	4	2	3	2	1	1	2	2	2	1	0	0
8	PEARL	3	2	3	2	0	2	2	3	2	2	2	2	2	2	1	2	1	1
9	RUTH	3	3	4	3	2	2	3	2	4	3	2	2	3	2	2	2	1	1
10	VERNE	2	2	3	2	1	1	2	2	3	4	3	3	4	3	3	2	1	1
11	MYRA	2	1	2	1	0	1	1	2	2	3	4	4	4	3	3	2	1	1
12	KATHERINE	2	1	2	1	0	1	1	2	2	3	4	6	6	5	3	2	1	1
13	SYLVIA	2	2	3	2	1	1	2	2	3	4	4	6	7	6	4	2	1	1
14	NORA	2	2	3	2	1	1	2	2	2	3	3	5	6	8	4	1	2	2
15	HELEN	1	2	2	2	1	1	2	1	2	3	3	3	4	4	5	1	1	1
16	DOROTHY	2	1	2	1	0	1	1	2	2	2	2	2	2	1	1	2	1	1
17	OLIVIA	1	0	1	0	0	0	0	1	1	1	1	1	1	2	1	1	2	2
18	FLORA	1	0	1	0	0	0	0	1	1	1	1	1	1	2	1	1	2	2

Evelyn and Theresa co-attended 7 events

Conversion to one-mode Data

- Projection using matrix multiplication
  - A^T x A gives the number of women who attended both the row event and the column event

		1	2	3	4	5	6	7	8	9	10	11	12	13	14
		E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14
1	E1	3	2	3	2	3	3	2	3	1	0	0	0	0	0
2	E2	2	3	3	2	3	3	2	3	2	0	0	0	0	0
3	E3	3	3	6	4	6	5	4	5	2	0	0	0	0	0
4	E4	2	2	4	4	4	3	3	3	2	0	0	0	0	0
5	E5	3	3	6	4	8	6	6	7	3	0	0	0	0	0
6	E6	3	3	5	3	6	8	5	7	4	1	1	1	1	1
7	E7	2	2	4	3	6	5	10	8	5	3	2	4	2	2
8	E8	3	3	5	3	7	7	8	14	9	4	1	5	2	2
9	E9	1	2	2	2	3	4	5	9	12	4	3	5	3	3
10	E10	0	0	0	0	0	1	3	4	4	5	2	5	3	3
11	E11	0	0	0	0	0	1	2	1	3	2	4	2	1	1
12	E12	0	0	0	0	0	1	4	5	5	5	2	6	3	3
13	E13	0	0	0	0	0	1	2	2	3	3	1	3	3	3
14	E14	0	0	0	0	0	1	2	2	3	3	1	3	3	3

14 women attended event E8 and one woman attended both Events E8 and E11

Two-mode Data

- Detection of overlapping communities
  - Crampes et Plantié, 2012
  - Only the concepts of the first two layers of the concept lattice are generated
  - Measures such as cohesion, separation and autonomy are used to define communities from concepts

Biclustering

- Dual-projection approach (Everett & Borgatti, 2012)

Core events (9E, 8E, 7E)

Peripheral events

		11E	12E	10E	14E	13E	9E	8E	7E	2E	3E	1E	5E	6E	4E
1	EVELYN						1	1		1	1	1	1	1	1
2	LAURA							1	1	1	1	1	1	1	
3	THERESA						1	1	1	1	1		1	1	1
4	BRENDA							1	1		1	1	1	1	1
5	CHARLOTTE								1		1		1		1
6	FRANCES							1			1		1	1	
7	ELEANOR							1	1				1	1	
8	PEARL						1	1						1	
9	RUTH						1	1	1				1		
10	VERNE		1				1	1	1						
11	MYRNA		1	1			1	1							
16	DOROTHY						1	1							
17	OLIVIA	1					1								
18	FLORA	1					1								
15	HELEN	1	1	1				1	1						
12	KATHERINE		1	1	1	1	1	1							
13	SYLVIA		1	1	1	1	1	1	1						
14	NORA	1	1	1	1	1	1		1					1	

Women in this group are structurally equivalent to core events (rows 8, 9, 10, 11, 16, 17, 18)

Three-mode Data

- How FCA can be helpful?
  - **Triadic concept analysis** (Lehmann & Wille, 1995)
    - Triadic contexts, concepts and diagrams
    - Concept trilattices and their visualization
  - **Triadic implications** (Biedermann, 1998)
  - **Polyadic concept analysis** (Voutsadakis, 2002)

Three-mode Data

- More recent work

  - Different types of **triadic implications** and **research topics** to explore (Ganter & Obiedkov, 2004)
  - Algorithm TRIAS for **triadic concept generation** (Jäschke et al., 2006)
  - Two algorithms for **triadic concept generation**: RSM and Cube Miner (Ji et al., 2006)
  - Data Peeler for *n*-**set computation** (Cerf et al., 2008)
  - **Inter-dimensional rules** (Nguyen et al., 2010)
  - **Triadic concept analysis** with **fuzzy** attributes (Belohlávek et al., 2010)

Triadic Concept Analysis

A **triadic context** $\mathbb{K} := (K_1, K_2, K_3, Y)$ where $Y \subseteq K_1 \times K_2 \times K_3$. The elements of K_1, K_2 and K_3 are called (formal) **objects**, **attributes** and **conditions**, respectively.
A triple (a_1, a_2, a_3) in Y means that object a_1 has attribute a_2 under condition a_3.

Triadic concept or (closed) tri-set

It is a triple (A_1, A_2, A_3) with $A_1 \subseteq K_1$, $A_2 \subseteq K_2$, $A_3 \subseteq K_3$ and $A_1 \times A_2 \times A_3 \subseteq Y$ such that no A_i (for i=1, 3) can be augmented without violating this condition. The subsets A_1, A_2 and A_3 are called the **extent**, the **intent** and the **modus** of the triadic concept (A_1, A_2, A_3) respectively.

Three-mode Data

- Three mode (tridimensional) data: objects, attributes and conditions
- E.g. events (1 .. 5), researchers (P, N, R, K, S) and roles (a, b, c, d)

$\mathbb{K}$	P	N	R	K	S
1	abd	abd	ac	ab	a
2	ad	bcd	abd	ad	d
3	abd	d	ab	ab	a
4	abd	bd	ab	ab	d
5	ad	ad	abd	abc	a

$\mathbb{K}^{(1)}$	P				N				R				K				S			
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d
1	1	1		1	1	1		1	1		1		1	1			1			
2	1			1		1	1	1	1	1		1	1			1				1
3	1	1		1				1	1	1			1	1			1			
4	1	1		1		1		1	1	1			1	1						1
5	1			1	1			1	1	1		1	1	1	1		1			

Three-mode Data

- Triadic concepts:
  - (12345, PRK, a), (12345, P, ad), (14,PN, bd), ...
- Rules
  - Any role (e.g., event organizer) played by S is also played by P
  - Whenever N attends events as a speaker (a) and PC member (d), then P does so

$\mathbb{K}$	P	N	R	K	S
1	abd	abd	ac	ab	a
2	ad	bcd	abd	ad	d
3	abd	d	ab	ab	a
4	abd	bd	ab	ab	d
5	ad	ad	abd	abc	a

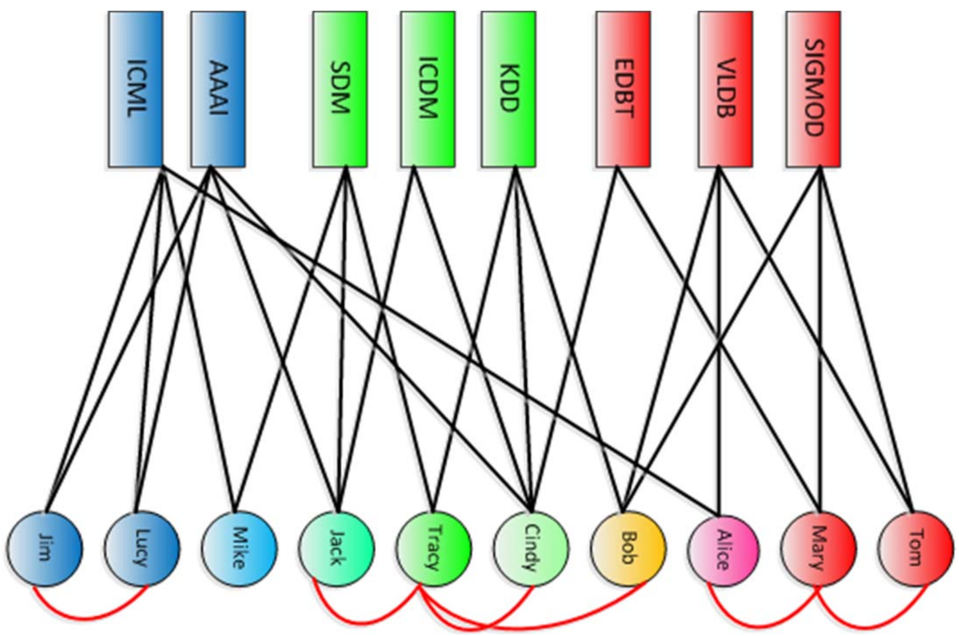
$\mathbb{K}^{(1)}$	P				N				R				K				S			
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d
1	1	1		1	1	1		1	1		1		1	1			1			
2	1			1		1	1	1	1	1		1	1			1				1
3	1	1		1				1	1	1			1	1			1			
4	1	1		1		1		1	1	1			1	1						1
5	1			1	1			1	1	1		1	1	1	1		1			

Heterogeneous information Networks

(Sun & Han, 2012)

Definition 1.1 (Information network) An *information network* is defined as a directed graph $G = (\mathcal{V}, \mathcal{E})$ with an object type mapping function $\tau : \mathcal{V} \rightarrow \mathcal{A}$ and a link type mapping function $\phi : \mathcal{E} \rightarrow \mathcal{R}$, where each object $v \in \mathcal{V}$ belongs to one particular object type $\tau(v) \in \mathcal{A}$, each link $e \in \mathcal{E}$ belongs to a particular relation $\phi(e) \in \mathcal{R}$, and if two links belong to the same relation type, the two links share the same starting object type as well as the ending object type.

Heterogeneous IN when the number of object or link types is >1



Conclusion

- Observations
  - Many studies in FCA can be usefully exploited for mining social networks: negation, ontology-based analysis, visualization(e.g., nested line diagrams), context transformation and decomposition, …
- Usefulness of FCA extensions
  - Triadic concept analysis (Lehmann & Wille 1995)
  - Logical CA (Ferré, 2000)
  - Relational CA (Rouane-Hacene et al., 2012),
  - Rough CA, fuzzy CA, etc.

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