ACTION RULES & META ACTIONS

presented by

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Grant I.
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An action rule /Ras & Wieczorkowska, PKDD 2000/ is a rule extracted from an information system that describes a possible transition of objects from one state to another with respect to a distinguished attribute called a decision attribute.

Assumption:
- attributes are partitioned into stable and flexible.

Values of attributes can be changed.
Introduction: **Action Rule**

Action rule is defined as a term \([(\omega) \land (\alpha \rightarrow \beta)] \rightarrow (\phi \rightarrow \psi)\)

Information System

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>b2</td>
<td>d1</td>
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<tr>
<td>a2</td>
<td>b2</td>
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<tr>
<td></td>
<td>a2</td>
<td>b2</td>
<td>d2</td>
</tr>
</tbody>
</table>
Example

Decision Table

<table>
<thead>
<tr>
<th>X</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁</td>
<td>0</td>
<td>S</td>
<td>0</td>
<td>L</td>
</tr>
<tr>
<td>x₂</td>
<td>0</td>
<td>R</td>
<td>1</td>
<td>L</td>
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<td>x₃</td>
<td>0</td>
<td>S</td>
<td>1</td>
<td>L</td>
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<tr>
<td>x₄</td>
<td>0</td>
<td>R</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>x₅</td>
<td>2</td>
<td>P</td>
<td>2</td>
<td>L</td>
</tr>
<tr>
<td>x₆</td>
<td>2</td>
<td>P</td>
<td>2</td>
<td>L</td>
</tr>
<tr>
<td>x₇</td>
<td>2</td>
<td>S</td>
<td>2</td>
<td>H</td>
</tr>
</tbody>
</table>

{a, c} - stable attributes,
{b, d} - flexible attributes,
d - decision attribute.

Rules discovered:
\[ r₁ = (b, P) \rightarrow (d, L) \]
\[ r₂ = (a, 2) \land (b, S) \rightarrow (d, H) \]

Action rule:
\[ [(a, 2) \land (b, P \rightarrow S)](x) \Rightarrow [(d, L \rightarrow H)](x) \]
Application domain: Customer Attrition

Facts:

- On average, most US corporations lose half of their customers every five years (Rombel, 2001).
- Longer a customer stays with the organization, the more profitable he or she becomes (Pauline, 2000; Hanseman, 2004).
- The cost of attracting new customers is five to ten times more than retaining existing ones.
- About 14% to 17% of the accounts are closed for reasons that can be controlled like price or service (Lunt, 1993).

Action:

Reducing the outflow of the customers by 5% can double a typical company’s profit (Rombel, 2001).
Customer Attrition in Retail Banking: US, Canada, UK, France

Report Published by Celent

Fighting customer attrition is a top priority at US and Canadian banks, and for good reason: customer defection rates are up to 7 times higher in the US and Canada than in Western Europe. A new Celent report explains why.

In a new report, “Customer Attrition in Retail Banking: the US, Canada, the UK, and France,” Celent analyzes why customer defection rates are so much higher in the US and Canada than the UK and France.

The report, which is based on a survey of more than 30 banks, reviews trends affecting customer attrition in the UK and France, analyzes how Canadian and American bankers have addressed attrition so far, and reports on their success. The report also suggests tactics that North American banks should pursue to get a 10% defection rate or better.

Over the past two years, half of top US and Canadian banks have seen their defection rates decrease by an average of 10%, while defection rates have remained the same for 40%. Only 10% have experienced increased attrition. The best organizations in the US and Canada have achieved a 12% customer defection rate.
Methods for Action Rules Extraction:

1] Rule-based
   Prior extraction of classification rules is needed
   Example – DEAR /Tsay & Ras, tree-based strategy/

2] Object-based
   Action rules are extracted directly from DB
   Example – ARED /similar to Apriori/

Ref 1: “Action rules discovery: System DEAR2, method and experiments”,
L.-S. Tsay, Z.W. Ras, Journal of Experimental and Theoretical Artificial Intelligence,
Taylor & Francis, Vol. 17, No. 1-2, 2005, 119-128

Ref 2: “Association Action Rules“, Z.W. Ras, A. Dardzinska, L.-S. Tsay, H. Wasyluk,
IEEE/ICDM Workshop on Mining Complex Data (MCD 2008), Pisa, Italy,
ICDM Workshops Proceedings, IEEE Computer Society, 2008, 283-290
Partition decision table $S$

Splitting the node using the stable attribute
$\text{Dom}(a) = \{1,2,3\}$ & $\text{Dom}(b) = \{1,2,3,4,5\}$

Stable: $\{a, b\}$
Flexible: $\{c, e, f\}$
Reclassification direction: $2 \rightarrow 1$ or $3 \rightarrow 1$

All objects have the same value 8 for attribute $f$, so it is crossed out from the sub-table (this condition is used for stable attributes as well)

None of the objects contain the desired class “1”, so this sub-table stops splitting any further

All the flexible values are the same for both objects, therefore this sub-table is not analyzed any further
DEAR - Rule-Based Action Rules Discovery

Set of rules $R$ with supporting objects

<table>
<thead>
<tr>
<th>Objects</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
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<tbody>
<tr>
<td>x1, x2, x3, x4</td>
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<td>L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x1, x3</td>
<td>0</td>
<td>L</td>
<td></td>
<td></td>
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<tr>
<td>x2, x4</td>
<td>2</td>
<td>L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2, x4</td>
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</tr>
<tr>
<td>x5, x6</td>
<td>3</td>
<td>L</td>
<td></td>
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<tr>
<td>x7, x8</td>
<td>2</td>
<td>1</td>
<td>H</td>
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<tr>
<td>x7, x8</td>
<td>1</td>
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<td>H</td>
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</table>

$(d, L)$-tree $T_1$

<table>
<thead>
<tr>
<th>Objects</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>x7, x8</td>
<td>2</td>
<td>1</td>
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</tr>
<tr>
<td>x7, x8</td>
<td>1</td>
<td>2</td>
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</tbody>
</table>

Stable Attribute: $\{a, c\}$
Flexible Attribute: $b$
Decision Attribute: $d$

$(d, H)$-tree $T_2$

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<tr>
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<tr>
<td>x1, x3</td>
<td>0</td>
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<tr>
<td>x2, x4</td>
<td>2</td>
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<tr>
<td>x2, x4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x5, x6</td>
<td>3</td>
<td></td>
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</tbody>
</table>

$(T_3, T_1): (a = 2) \land (b, 2 \rightarrow 1) \Rightarrow (d, L \rightarrow H)$
$(a = 2) \land (b, 3 \rightarrow 1) \Rightarrow (d, L \rightarrow H)$
ARED - Object Based Action Rule Discovery

Decision System $S$

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$sup(r) = 2$

$conf(r) = 2/2 = 1$

$(a, a_1 \rightarrow a_1)$  $(a, a_1)\quad Y = \{x_2, x_4\}$
$(a, a_2 \rightarrow a_2)$  $(a, a_2)\quad Z = \{x_1, x_2, x_3, x_4, x_5, x_7\}$
$(b, b_1 \rightarrow b_1)$  $(b, b_1)\quad$ atomic action terms
$(b, b_2 \rightarrow b_2)$

$\cdots\cdots$.

$(d, d_1 \rightarrow d_1)$
$(d, d_2 \rightarrow d_2)$

$r = [(a, a_2 \rightarrow a_2)*(b, b_1 \rightarrow b_1)] \rightarrow (d, d_1 \rightarrow d_1)$

$sup(r) = \text{card}(Y \cap Z)$

$conf(r) = \left[ \frac{\text{card}(Y \cap Z)}{\text{card}(Y)} \right]$
### Decision System $S$

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- $(a, a_1 \rightarrow a_1)$
- $(a, a_1 \rightarrow a_2)$
- $(b, b_1 \rightarrow b_2)$
- $(b, b_2 \rightarrow b_2)$
- $\ldots\ldots\ldots$
- $(d, d_1 \rightarrow d_1)$
- $(d, d_2 \rightarrow d_2)$

$r = [(a, a_2 \rightarrow a_1)^{*}(b, b_1 \rightarrow b_1)] \rightarrow (d, d_1 \rightarrow d_2)$

$\sup(r) = 2$

$\text{conf}(r) = 1/2$

$Y_1 = \{x_2, x_4\}$

$Z_1 = \{x_1, x_2, x_3, x_4, x_5, x_7\}$

$Y_2 = \{x_1, x_6\}$

$Z_2 = \{x_6\}$

$\sup(r) = \text{card}(Y_1 \cap Z_1)$

$\text{conf}(r) = \left[\frac{\text{card}(Y_1 \cap Z_1)}{\text{card}(Y_1)}\right] \times \left[\frac{\text{card}(Y_2 \cap Z_2)}{\text{card}(Y_2)}\right]$
### Decision System $S$

<table>
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</table>

- $(a, a_1 \rightarrow a_1)$
- $(a, a_1 \rightarrow a_2)$
- $(b, b_1 \rightarrow b_2)$
- $(b, b_2 \rightarrow b_2)$
- $(d, d_1 \rightarrow d_1)$
- $(d, d_2 \rightarrow d_2)$

\[
\begin{align*}
Y_1 &= \{x_2, x_4\} \\
Z_1 &= \{x_1, x_2, x_3, x_4, x_5, x_7\} \\
Y_2 &= \{x_1, x_6\} \\
Z_2 &= \{x_6\}
\end{align*}
\]

\[
\begin{align*}
sup(r) &= 1 \\
conf(r) &= 1/2
\end{align*}
\]

$sup(r) = \min\{\text{card}(Y_1 \cap Z_1), \text{card}(Y_2 \cap Z_2)\}$

\[
conf(r) = \left[\frac{\text{card}(Y_1 \cap Z_1)}{\text{card}(Y_1)}\right] * \left[\frac{\text{card}(Y_2 \cap Z_2)}{\text{card}(Y_2)}\right]
\]
**ARED**

\[ \lambda_1 - \text{minimum support, } \lambda_2 - \text{minimum confidence} \]

Object reclassification from class \(d_1\) to \(d_2\)  \(\lambda_1=2, \lambda_2=1/4\)

---

Meaning of \((d, d_1 \rightarrow d_2)\) in \(S\): \[ N_S(d,d_1 \rightarrow d_2)=[\{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_6\}] \]

Atomic classification terms:

- \((b,b_1 \rightarrow b_1), (b,b_2 \rightarrow b_2), (b,b_3 \rightarrow b_3)\)
- \((a,a_1 \rightarrow a_2), (a,a_1 \rightarrow a_1), (a,a_2 \rightarrow a_2), (a,a_2 \rightarrow a_1)\)
- \((c,c_1 \rightarrow c_2), (c,c_2 \rightarrow c_1), (c,c_1 \rightarrow c_1), (c,c_2 \rightarrow c_2)\)

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<table>
<thead>
<tr>
<th>(X)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
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<tr>
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</tbody>
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*stable attribute*

*flexible attributes*
Object reclassification from class $d_1$ to $d_2$  \hspace{1cm} \lambda_1=2, \ \lambda_2=1/4

Notation:

$t_1=(b,b_1 \rightarrow b_1), \ t_2=(b,b_2 \rightarrow b_2), \ t_3=(b,b_3 \rightarrow b_3), \ t_4=(a,a_1 \rightarrow a_2), \ t_5=(a,a_1 \rightarrow a_1), \ t_6=(a,a_2 \rightarrow a_2), \ t_7=(a,a_2 \rightarrow a_1), \ t_8=(c,c_1 \rightarrow c_2), \ t_9=(c,c_2 \rightarrow c_1), \ t_{10}=(c,c_1 \rightarrow c_1), \ t_{11}=(c,c_2 \rightarrow c_2), \ t_{12} = (d,d_1 \rightarrow d_2).$
Object reclassification from class $d_1$ to $d_2$ \( \lambda_1=2, \lambda_2=1/4 \)

For decision attribute in $S$:

\[
N_S(d,d_1 \rightarrow d_2) = \{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_6\}
\]

For classification attribute in $S$:

\[
\sup(r) = \text{card}(Y_1 \cap Z_1)
\]

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$a$</th>
<th>$b$</th>
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</tbody>
</table>

Not marked $\lambda_1=3$

Mark “-” $\lambda_2=0$

Mark “-” $\lambda_1=1$

Mark “-” $\lambda_2=0$
Object reclassification from class $d_1$ to $d_2$ \[ \lambda_1=2, \lambda_2=1/4 \]

<table>
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<td>$b_2$</td>
<td>$c_1$</td>
<td>$d_3$</td>
</tr>
</tbody>
</table>

For decision attribute in $S$: $N_S(d, d_1 \rightarrow d_2)=[\{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_6\}]$

For classification attribute in $S$:

- Not marked \[
\lambda_1=2
\]
- Mark "-" \[
\lambda_2=0
\]
- Mark "+") \[
\lambda_1=4, \lambda_2=1/4
\]

$N_S(t_5) = N_S(a, a_1 \rightarrow a_1) = [\{x_1, x_6, x_7, x_8\}, \{x_1, x_6, x_7, x_8\}]$

$N_S(t_6) = N_S(a, a_2 \rightarrow a_2) = [\{x_2, x_3, x_4, x_5\}, \{x_2, x_3, x_4, x_5\}]$

$N_S(t_7) = N_S(a, a_2 \rightarrow a_1) = [\{x_2, x_3, x_4, x_5\}, \{x_1, x_6, x_7, x_8\}]$
Object reclassification from class $d_1$ to $d_2$

$\lambda_1=2, \, \lambda_2=1/4$

For decision attribute in $S$:

$N_S(t_{12})=\{\{x_1,x_2, x_3, x_4, x_5, x_7\}, \{x_6\}\}$

For classification attribute in $S$:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$a$</th>
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<th>$c$</th>
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<tr>
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<tr>
<td>$x_8$</td>
<td>$a_1$</td>
<td>$b_2$</td>
<td>$c_1$</td>
<td>$d_3$</td>
</tr>
</tbody>
</table>

Marked “-” $\lambda_2=0$

| $N_S(t_1)=$| $\{\{x_1,x_2, x_4, x_6\}, \{x_1,x_2, x_4, x_6\}\}$ |
| $N_S(t_2)=$| $\{\{x_3,x_7, x_8\}, \{x_3,x_7, x_8\}\}$ |
| $N_S(t_3)=$| $\{\{x_5\}, \{x_5\}\}$ |
| $N_S(t_4)=$| $\{\{x_1,x_6, x_7, x_8\}, \{x_2,x_3, x_4, x_5\}\}$ |
| $N_S(t_5)=$| $\{\{x_1,x_6, x_7, x_8\}, \{x_1,x_6, x_7, x_8\}\}$ |
| $N_S(t_6)=$| $\{\{x_2,x_3, x_4, x_5\}, \{x_2,x_3, x_4, x_5\}\}$ |
| Mark “+” $\lambda_1=4, \, \lambda_2=1/4$ | $N_S(t_7)=$ | $\{\{x_2,x_3, x_4, x_5\}, \{x_1,x_6, x_7, x_8\}\}$ |

$r = [t_7 \rightarrow t_1]$
Object reclassification from class \( d_1 \) to \( d_2 \)
\( \lambda_1 = 2, \quad \lambda_2 = \frac{1}{4} \)

For decision attribute in \( S \):
\[
N_S(t_{12}) = \left[ \{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_6\} \right]
\]

For classification attribute in \( S \):
\[
\text{conf} = \frac{2}{3} \times \frac{1}{5} < \lambda_2
\]
Object reclassification from class $d1$ to $d2$ \[ \lambda_1=2, \lambda_2=1/4 \]

Now action terms of length 2 from unmarked action terms of length 1

For decision attribute in $S$:
\[ N_s(t_{12})=\{\{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_6\}\} \]

For classification attribute in $S$:
\[ N_s(t_1)=\{\{x_1, x_2, x_4, x_6\}, \{x_1, x_2, x_4, x_6\}\} \]
\[ N_s(t_5)=\{\{x_1, x_6, x_7, x_8\}, \{x_1, x_6, x_7, x_8\}\} \]
\[ N_s(t_8)=\{\{x_1, x_4, x_8\}, \{x_2, x_3, x_5, x_6, x_7\}\} \]
\[ N_s(t_{11})=\{\{x_2, x_3, x_5, x_6, x_7\}, \{x_2, x_3, x_5, x_6, x_7\}\} \]

Marked “-”, $\lambda_1=1$

\[ N_s(t_1^{*}t_5)=[\{x_1, x_6\}, \{x_1, x_6\}] \]

Marked “+”

\[ N_s(t_1^{*}t_8)=[\{x_1, x_4\}, \{x_2, x_6\}] \]

Rule \( r = [t_1^{*}t_8 \rightarrow t_{12}] \), conf = 1/2 ≥ $\lambda_2$, sup=2 ≥ $\lambda_1$
Object reclassification from class $d_1$ to $d_2$  \( \lambda_1=2, \ \lambda_2=1/4 \)

For decision attribute in $S$:

\[
N_5(t_{12})=\{ \{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_6\} \}
\]

For classification attribute in $S$:

**Action rules:**

\[
[[ (b, b_{1}\rightarrow b_{1}) \ast (c, c_{1}\rightarrow c_{2}) ] \rightarrow (d, d_{1}\rightarrow d_{2}) ]
\]

\[
[[ (a, a_{2}\rightarrow a_{1}) \rightarrow (d, d_{1}\rightarrow d_{2}) ]
\]
Association Action Rules

Atomic Action Terms:

1. \((a, a_1 \rightarrow a_2), \text{ where } a \text{ is symbolic attribute}\)

\[a_1, a_2 \in V_a\]

if \(a_1 = a_2\) then \(a\) - stable on \(a_1\)

2. \((a, [a_1, a_2] \uparrow [a_3, a_4]), \text{ where } a \text{ is numerical, } a_1 \leq a_2 < a_3 \leq a_4, \text{ and } a_i \in V_a\)

3. \((a, [a_1, a_2] \downarrow [a_3, a_4]), \text{ where } a \text{ is numerical, } a_1 \leq a_2 < a_3 \leq a_4, \text{ and } a_i \in V_a\)

\(\uparrow\) - value is increased;  \(\downarrow\) - value is decreased
Candidate action set - any collection of atomic actions

Action set - candidate action set which does not contain two atomic actions referring to the same attribute

Example: \[\{(b, b_2), (b, [b_1, b_2] \uparrow [b_3, b_4])\}\] – candidate action set but not action set.

Action term - conjunction of atomic actions forming an action set.

Example: \((a, a_2) \land (c, c_1 \rightarrow c_2) \land (b, [b_1, b_2] \uparrow [b_3, b_4])\)
Association Action Rules

Interpretation $N_S$ of action terms in $S=(X,A,V)$:

$$N_S((a, a_1 \rightarrow a_2)) = \{x \in X : a(x) = a_1\}, \{x \in X : a(x) = a_2\}.$$  

$$N_S((a, [a_1, a_2] \downarrow [a_3, a_4])) =$$  

$$\{\{x \in X : a_1 \leq a(x) \leq a_2\}, \{x \in X : a_3 \leq a(x) \leq a_4\}\}.$$  

$$N_S((a, [a_1, a_2] \uparrow [a_3, a_4])) =$$  

$$\{\{x \in X : a_1 \leq a(x) \leq a_2\}, \{x \in X : a_3 \leq a(x) \leq a_4\}\}.$$  

$$N_S((a, \uparrow [a_3, a_4])) =$$  

$$\{\{x \in X : a(x) < a_3\}, \{x \in X : a_3 \leq a(x) \leq a_4\}\}.$$  

$$N_S((a, [a_3, a_4] \uparrow)) =$$  

$$\{\{x \in X : a_3 \leq a(x) \leq a_4\}, \{x \in X : a_4 < a(x)\}\}.$$
Association Action Rules

Information System $S=(X, A, V)$

Action Rule: $r = [t \rightarrow t']$ \hspace{1cm} $t = t_1 \land t_2 \land \ldots \land t_k$

$N_S(t) = [Y_1, Y_2]$ \hspace{1cm} $N_S(t') = [Z_1, Z_2]$

Support: $\text{sup}(r) = \min\{\text{card}(Y_1 \cap Z_1), \text{card}(Y_2 \cap Z_2)\}$

Confidence: $\text{conf}(r) = \left[\frac{\text{card}(Y_1 \cap Z_1)}{\text{card}(Y_1)}\right] \cdot \left[\frac{\text{card}(Y_2 \cap Z_2)}{\text{card}(Y_2)}\right]$
**Meta-actions /A. Tuzhilin (2006)/**

Actions which trigger changes of flexible attributes either directly or indirectly because of correlations among certain attributes in the system.

**Example 1:** Taking a drug /Lamivudine is used for treatment of chronic hepatitis B. It *improves the seroconversion of e-antigen positive hepatitis B* and also *improves histology staging of the liver* but at the same time it can cause a number of other symptoms. This is why doctors have to order certain lab tests to check patient’s response to that drug.

**Example 2:** Classification attributes are: *Explain difficult concepts effectively, Stimulate student interest in the course, Provide sufficient feedback.* Meta-actions: *Change the content of the course, Change the textbook of the course, ...*
Action Rules - continuation

**Influence Matrix**

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>.....</th>
<th>$A_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
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<td>$E_{12}$</td>
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<td>$E_{14}$</td>
<td></td>
<td>$E_{1m}$</td>
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<tr>
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<tr>
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<td>$E_{31}$</td>
<td>$E_{32}$</td>
<td>$E_{33}$</td>
<td>$E_{34}$</td>
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<td>$E_{3m}$</td>
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<tr>
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<td>$E_{m3}$</td>
<td>$E_{m4}$</td>
<td></td>
<td>$E_{mn}$</td>
</tr>
</tbody>
</table>

$A_1, A_2, A_3, ..., A_m$ – attributes
$M_1, M_2, M_3, ..., M_n$ – meta actions
$E_{ij}$ – atomic action or NULL, $i \leq m, j \leq n$

**Atomic actions triggered by $M_3$**

**Action rule:** $r = [t \Rightarrow (d, d_1 \rightarrow d_2)]$, where $t$ – action term
Action Rules - continuation

Meta-actions based decision system

\[ S(d) = (X, A \cup \{d\}, V) \]

with \( A = \{A_1, A_2, \ldots, A_m\} \)

Influence Matrix

Candidate action rule -

\[ r = [(A_1, a_1 \rightarrow a_1'), (A_2, a_2 \rightarrow a_2'), (A_4, a_4 \rightarrow a_4')] \Rightarrow (d, d_1 \rightarrow d_1') \]

We say that \( r \) is valid in \( S \) with respect to meta-action \( M_3 \) if the atomic actions triggered by \( M_3 \) do not contradict with atomic actions in \( r \).
Meta-actions based decision system $S(d) = (X, A \cup \{d\}, V)$, with $A = \{A_1, A_2, \ldots, A_m\}$

<table>
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<td>$E_{m3}$</td>
<td>$E_{m4}$</td>
<td>.....</td>
<td>$E_{mn}$</td>
</tr>
</tbody>
</table>

We say that action rule $r$ extracted from $S(d)$ is supported by meta-actions in $M=\{M_{i1}, M_{i2}, \ldots, M_{in}\}$, if $r$ is valid in $S$ with respect to each meta-action in $M$ and each atomic action at the left-hand side of $r$ is triggered by minimum one meta-action in $M$. 
Action Rules Discovery - Example

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
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<tbody>
<tr>
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<td>b₁</td>
<td>c₂  → c₁</td>
</tr>
<tr>
<td>M₂</td>
<td>a₂  → a₁</td>
<td>b₂</td>
<td></td>
</tr>
<tr>
<td>M₃</td>
<td>a₁  → a₂</td>
<td></td>
<td>c₂  → c₁</td>
</tr>
<tr>
<td>M₄</td>
<td></td>
<td>b₁</td>
<td>c₁  → c₂</td>
</tr>
<tr>
<td>M₅</td>
<td></td>
<td></td>
<td>c₁  → c₂</td>
</tr>
<tr>
<td>M₆</td>
<td>a₁  → a₂</td>
<td></td>
<td>c₁  → c₂</td>
</tr>
</tbody>
</table>

Influence Matrix for S(d)

Candidate action rules:

\[ r₁ = [(b, b₁) \land (c, c₁ \rightarrow c₂)] \Rightarrow (d, d₁ \rightarrow d₂) \], valid with respect to M₄, M₅, M₆

\[ r₂ = [(a, a₂ \rightarrow a₁)] \Rightarrow (d, d₁ \rightarrow d₂) \], valid with respect to M₂, M₄, M₅ but not valid with respect to M₂
Application Domain

Database HEPAR (Medical Center of Postgraduate Education, Warsaw, Poland) prepared by Dr. med. Hanna Wasyluk

- 758 patients described by 106 attributes (including 31 laboratory tests with values discretized to: “below normal”, “normal”, “above normal”). 14 attributes are stable. Two tests are invasive tests: HBsAg [in tissue], HBcAg [in tissue]. There are 7 decision values (attribute d):

I. Acute hepatitis
IIa. Subacute hepatitis (types BC)
IIb. Subacute hepatitis (alcohol-abuse)
IIIa. Chronic hepatitis (curable)
IIIb. Chronic hepatitis (non-curable)
IV. Cirrhosis hepatitis
V. Liver cancer

We ask for re-classifications: IIb → I, IIIa → I
Application Domain

Chosen d-reduct has no invasive tests, 11% incomplete data

\{m, n, q, u, y, aa, ah, ai, am, an, aw, bb, bg, by, cj, cm\}

m  Bleeding
n  Subjaundice symptoms
q  Eructation
u  Obstruction
y  Weight loss
aa  Smoking
ah  History of viral hepatitis (stable)
ai  Surgeries in the past (stable)
am  History of hospitalization (stable)
an  Jaundice in pregnancy
aw  Erythematous dermatitis
bb  Cysts
bg  Sharp liver edge (stable)
bm  Blood cell plaque
by  Alkaline phosphatase
cj  Prothrombin index
cm  Total cholesterol
d  Decision attribute
Application Domain: Database HEPAR

- No history of viral hepatitis but with history of surgery and hospitalization, sharp liver edge normal, no subjaundice symptoms, total cholesterol normal, erythematous dermatitis normal, weight normal, no cysts, patient does not smoke

\[ ((ah=1) \land (ai=2) \land (am=2) \land (bg=1)) \land (cm=1) \land (aw=1) \land (u, \rightarrow I) \land (bb=1) \land (aa=1) \]
\[ \land (q, \rightarrow I) \land (m, \rightarrow I) \land (n=1) \land (bm, \rightarrow up) \land (y=1) \land (by, \rightarrow up) \]
\[ \rightarrow (d, IIIa \rightarrow I) \]

\[ ((ah=1) \land (ai=2) \land (am=2) \land (bg=1)) \land (cm=1) \land (aw=1) \land (u, \rightarrow I) \land (bb=1) \land (aa=1) \]
\[ \land (q, \rightarrow I) \land (m, \rightarrow I) \land (n=1) \land (bm, \rightarrow up) \land (y=1) \land (by, \rightarrow down) \]
\[ \rightarrow (d, IIIa \rightarrow I) \]

Two quite similar action rules have been constructed:

*By getting rid of obstruction, eructation, bleeding, by decreasing the blood cell plaque and by changing the level of alkaline phosphatase we should be able reclassify the patient from IIIa group to I.*

Attribute values of *total cholesterol, weight, and smoking* have to stay unchanged.
Meta Actions (drugs)

Action Rule II requires getting rid of:

- obstruction (u)  
  - HEPATIL  
  - TRIPHALA
- eructation (q)  
  - HEPATIL  
  - HEPARGEN
- bleeding (m)  
  - HEPATIL  
  - HEPARGEN
- decreased blood cell plaque (bm)
  - HEPATIL  
  - HEPARGEN
- increased level of alkaline phosphatase (by)
  - HEPATIL  
  - HEPARGEN

\[
\begin{align*}
[(ah=1) \land (ai=2) \land (am=2) \land (bg=1)] \land (cm=1) \land (aw=1) \land (u, \rightarrow I) \land (bb=1) \\
(aa=1) \land (q, \rightarrow I) \land (m, \rightarrow I) \land (n=1) \land (bm, \rightarrow up) \land (y=1) \land (by, \rightarrow \text{down}) \\
\rightarrow (d, \text{IIIa} \rightarrow I)
\end{align*}
\]

\[
\begin{align*}
[(ah=1) \land (ai=2) \land (am=2) \land (bg=1)] \land (cm=1) \land (aw=1) \land (bb=1) \\
(aa=1) \land (n=1) \land (y=1) \land \text{obstruction, eructation, bleeding, increased blood cell plaque, increased level of alkaline phosphatase } \land \text{[Take HEPATIL]} \rightarrow (d, \text{IIIa} \rightarrow I)
\end{align*}
\]

\[
\begin{align*}
[(ah=1) \land (ai=2) \land (am=2) \land (bg=1)] \land (cm=1) \land (aw=1) \land (bb=1) \land (aa=1) \land (n=1) \land (y=1) \land \text{obstruction, eructation, bleeding, increased blood cell plaque, increased level of alkaline phosphatase } \land \text{[Take HEPERGEN & TRIPHALA]} \rightarrow (d, \text{IIIa} \rightarrow I)
\end{align*}
\]
Simple Association Action Rules

\[(a, a_1 \rightarrow a_2)\] - atomic action set

\[cost((a, a_1 \rightarrow a_2))\] - cost of action expecting to change value of attribute \(a\) from \(a_1\) to \(a_2\).

\[t_1 = (a, a_1 \rightarrow a_2), \ t_2 = (b, b_1 \rightarrow b_2)\] - two atomic action sets

\(t_1, t_2\) are positively correlated if changes \(t_1, t_2\) support each other.
Simple Association Action Rules

Definition:

Let \( t = t_1 \cdot t_2 \cdot \ldots \cdot t_m \) is a frequent action set, where each \( t_i \) - atomic action set.

Let \( T = \{t_1, t_2, \ldots, t_m\} \) and:

\[ t_i \sim t_j \quad \text{iff} \quad t_i \text{ and } t_j \text{ are positively correlated.} \]

Equivalence relation, partitions \( T \) into equivalence classes \( (T = T_1 \cup T_2 \cup \ldots \cup T_k) \)
Simple Association Action Rules

Definition:

Let \( t = t_1 \cdot t_2 \cdot \ldots \cdot t_m \) is a frequent action set, where each \( t_i \) - atomic action set.

Let \( T = \{t_1, t_2, \ldots, t_m\} \) and:

\[ t_i \sim t_j \text{ iff } t_i \text{ and } t_j \text{ are positively correlated.} \]

Now:

In each equivalence class \( T_i \), an atomic action set \( a(T_i) \) of the lowest cost is identified.

The cost of \( t \) is defined as: \( \text{cost}(t) = \sum \{\text{cost}(a(T_i)): 1 \leq i \leq k\} \)
Definition:

Let $t = t_1 * t_2 * ... * t_m$ is a frequent action set, where each $t_i$ - atomic action set.

Let $T = \{t_1, t_2, ..., t_m\}$ and:

$t_i \sim t_j \text{ iff } t_i \text{ and } t_j \text{ are positively correlated.}$

Now:

The cost of $t$ is defined as: $cost(t) = \sum_{i=1}^{k} cost(a(T_i))$  

$a(T_1) * a(T_2) * ... * a(T_k) \rightarrow [t - \{a(T_i): 1 \leq i \leq k\}]$

- simple association action rule.
Simple Association Action Rules

Definition:

Let $t = t_1 \ast t_2 \ast \ldots \ast t_m$ is a frequent action set, where each $t_i$ - atomic action set.

Let $T = \{t_1, t_2, \ldots, t_m\}$ and:

$t_i \sim t_j$ iff $t_i$ and $t_j$ are positively correlated.

Now:

The cost of $t$ is defined as: $\text{cost}(t) = \sum \{\text{cost}(a(T_i)): 1 \leq i \leq k\}$

$r = [a(T_1) \ast a(T_2) \ast \ldots \ast a(T_k) \rightarrow [t - \{a(T_i): 1 \leq i \leq k\}]]$

- simple association action rule.

The cost of $r$ is defined as the cost of $a(T_1) \ast a(T_2) \ast \ldots \ast a(T_k)$
Some Publications


Questions?

Thank You